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Complex Systems as Lenses

on

Learning and Teaching

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Complex Systems as Lenses

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Learning and Teaching

by

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Dedication

This work is dedicated to my wife, Nola Charles Hurford, to my parents, Jack and Elizabeth Hurford, and to my sister, Lisa Anne Hurford.

PREFACE

This dissertation is arranged as a series of standalone articles. Section 1, titled: "What Complexity Theories Have to Say About Learning: A Review of the Literature" reviews the literature on complexity science and on learning theories and identifies intersections between research on complex and research on learning. Section 2, titled: "Order in chaos: Scale-invariant behaviors in Participatory Simulations" is an empirical research article that investigates the occurrence of scale invariance in learners' classroom activity. Section 3, titled: "Complex Systems as Content and Structure in a Learning Theories Course" is intended to inform teachers' practice and to begin to infuse a theory of learning as complex adaptive system in teacher preparation courses.

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From metaphors to mathematized models, the complexity sciences are changing the ways disciplines view their worlds, and ideas borrowed from complexity are increasingly being used to structure conversations and guide research on teaching and learning. The purpose of this corpus of research is to further those conversations and to extend complex systems ideas, theories, and modeling to curricula and to research on learning and teaching. A review of the literatures of learning and of complexity science and a discussion of the intersections between those disciplines are provided. The work

reported represents an evolving model of learning *qua* complex system and that evolution is the result of iterative cycles of design research.

One of the signatures of complex systems is the presence of scale invariance and this line of research furnishes empirical evidence of scale invariant behaviors in the activity of learners engaged in participatory simulations. The offered discussion of possible causes for these behaviors and chaotic phase transitions in human learning favors real-time optimization of decision-making as the means for producing such behaviors. Beyond theoretical development and modeling, this work includes the development of teaching activities intended to introduce pre-service mathematics and science teachers to complex systems. While some of the learning goals for this activity focused on the introduction of complex systems as a content area, we also used complex systems to frame perspectives on learning.

Results of scoring rubrics and interview responses from students illustrate attributes of the proposed model of complex systems learning and also how these pre-service teachers made sense of the ideas. Correlations between established theories of learning and a complex adaptive systems model of learning are established and made explicit, and a means for using complex systems ideas for designing instruction is offered. It is a fundamental assumption of this research and researcher that complex systems ideas and understandings can be appropriated from more complexity-developed disciplines and put to use modeling and building increasingly productive understandings of learning and teaching.

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SECTION 1

WHAT COMPLEXITY THEORIES HAVE TO SAY ABOUT LEARNING: A REVIEW OF THE LITERATURE

PROLOGUE

Systems approaches that try to understand experience generally by looking at multiple sources of activity or interest and the relationships between them are becoming increasingly prevalent and important in widely disparate disciplines. The purpose of this paper is to undertake a discussion of the use of systems-theoretical approaches, particularly "dynamical systems" theories, for making sense of learning. The article begins with a general review of the development of systems-theoretical perspectives and then explores three of these in detail. These sections are followed by a consideration of some of the affordances of systems approaches for efforts at understanding learning.

GENERAL SYSTEMS APPROACHES

The use of systems-theoretical approaches for trying to understand experience is not new (van Gelder & Port, 1995, p. 4). As has been pointed out by Chen and Stroup (1993), Aristotle's "whole is greater than the sum of the parts" is perhaps the oldest recorded axiom of systems theory, but it seems reasonable that thinking about aggregates of "agents" (geese or cattle or armies) creating global patterns (Vs or stampedes or battles) predates Aristotle by quite a bit. From the ancient, to modern, to the present, the use and development of these types of stances have been recurrent events. What *is* relatively new however is an interdisciplinary effort toward developing a mathematized formalization of general systems-theoretical phraseology and methods into something approaching what John Casti (1994) refers to as a "science of surprise" (p. 15).

The bulk of modern scientific and mathematical thinking for the past several hundred years has been predominantly about dividing experience and phenomena into increasingly smaller parts in efforts to understand our world. However, over about the last half century, when human enterprise has turned to describing, explaining, or predicting dynamical activities, various strains of (holistic) systems-mathematical approaches have sometimes been invoked. Chen and Stroup (1993, p. 449) credit Lotka (1920/1956) with the establishment of the underpinnings of a science of systems based on his pioneering work in biology. Ludwig von Bertalanffy is probably the "father" of modern systems theory, publishing his seminal volume, *General Systems Theory* in 1968. Jay Forrester published Principles of Systems (1968) at about the same time. These two books and authors have gone a long way toward establishing a *science* of systems, laying out a cohesive framework fundamental terminology, categories, and processes of generalized systems perspectives and explicating their ideas with examples from the physical, biological, and social fields (Bertalanffy, 1968, chap. 1).

As mentioned above, Bertalanffy and Forrester are credited with building a "'new' field of study" (Chen & Stroup, 1993, p. 451), much more recently other authors have published systems-theoretical points of view (e.g., Casti, 1994; Camazine, 2001; Clark, 1997; Holland, 1995, 1998; Prigogine and Stengers, 1984, 1997), and the field is still rapidly expanding. Most notably for the purposes of this article, Jean Piaget developed major portions of his theories of human development around the idea of the algebraic group, "A mathematical group is a system consisting of a set of elements (e.g., the integers, positive and negative) together with an operation or rule of combination" (Piaget, 1968/1970, p. 18). He goes on to clarify the relationship between elements and operations:

> If the character of structured wholes depends on their laws of composition, these laws must of their very nature be *structuring*: it is the constant duality, or

> bipolarity, of always being *structuring* and *structured* that accounts for the success of the notion of law or rule employed by structuralists. …a structure's laws of composition are defined "implicitly," i.e., as governing the transformations of the system they structure. (p. 10)

Piaget's mutuality between the elements of the group and the operations that structure it correlates nicely with Bertalanffy's (1968) definition of system:

> A system can be defined as a set of elements standing in interrelations. Interrelation means that elements, *p*, stand in relations *R*, so that the behavior of an element *p* in *R* is different from its behavior in another relation, *R′*. (p. 55-56)

Viewing human development and learning in terms congruent with a systems-theoretical perspective, at least insofar as Piaget's meanings of system and group and Bertalanffy's are decidedly similar, reaches back to Piaget's work in the middle part of the last century.

Bertalanffy (1968) and Forrester (1968) established the fundamentals of general systems-theoretical approaches, but many others have published works that further help to formalize and increase the meanings and potential utility of systems perspectives (Camazine, et al., 2001; Casti, 1994; Clark, 1997; Holland, 1995, 1998; Kauffman, 1995; Prigogine & Stengers, 1984, 1997). There is also an increasing number of academic centers and institutes devoted to the study of complex systems: the Santa Fe Institute, Argonne National Labs, International Solvay Institutes for Physics and Chemistry, and the New England Complex Sciences Institute[1] are representative of the levels of national and international interest in these new approaches for developing disciplinary and interdisciplinary systems understandings. Although a thorough discussion would go beyond the scope of this paper, it seems reasonable to assert that large-scale Kuhnian (1962) paradigm shifts towards systems-theoretical thinking are taking place in a very wide range of disciplinary fields. No less an authority than Stephen Hawking has been quoted as saying "I think the next century will be the century of complexity" (Davis & Simmt, 2003, p. 137).

For example, there has been a huge expansion in discussions and implementation of systems-theoretical perspectives in almost every corner of the educational research community. From the recently formed "Complexity Special Interest Group" at the annual meeting of the American Education Research Association to the International Society of Learning Sciences to the International Group for the Psychology of Mathematics Education, complexity and "systems" are beginning to find their ways into much of the thinking and theorizing on schooling and learning. Although much of the work being done on complexity and learning is of very high quality (Ennis, 1992; Kieren & Simmt, 2002; Thelen & Smith, 1996; Wilensky & Resnick, 1999), there is also a tendency in the field to promote work that is vague, sensationalizing, or poorly considered. This is unfortunate because popularized and sensationalized discussions of complexity do very little good in furthering understandings of teaching, learning, and schooling, and they may do quite a bit of harm in terms of diminishing the credibility of the application of systems-theoretical perspectives to education. We share John Casti's (1994, p. 270) hopes for more theoretically grounded, mathematized and formalized approaches in the application of systems theories in general and to educational research in particular.

There is an important distinction to be made in a discussion of systems-theoretical approaches and education. Two general perspectives can be found in the literature—one is related to teaching and learning *about* complex systems (e.g., Resnick & Wilensky, 1998), the other is related to learning *as* a complex system (e.g., Ennis, 1992; Hurford, 1998; Thelen & Smith, 1996). Although "learning about" and "learning as" are apt to be mutually informative and learning about complexity and systems analyses is arguably an increasingly important candidate for school curricula, the primary focus in this discussion is on learning *as* a complex system.

In-Depth View of Three Systems Perspectives

The purpose of this section is to discuss in some detail three different systems-theoretical perspectives. We begin with John Casti's (1994) book, *Complexification: Explaining a Paradoxical World Through the Science of Surprise*, and follow that with a discussion of Camazine, et al. (2001), *Self-organization in Biological Systems*, and end the section with John Holland's (1995) *Hidden order: How Adaptation Builds Complexity*. This order of discussion has been chosen because it represents increasing levels of specificity and applicability to systems of learners–that is, to classrooms[2]. Casti's book identifies several familiar "complexity generators" and describes the necessity of developing new modeling tools for the purposes of understanding complex systems. This article then turns to Camazine et al., in order to consider one "formalized" approach at understanding how complex patterns emerge from the activities of multi-agent systems, and finally treats Holland's work because it seems to do the best job of rising to the closing challenge in Casti's *Complexification:*

> For complexity to become a science, it is necessary–but far from sufficient–to formalize our intuitive notions about complexity in symbols and syntax… the creation of a science of complex systems is really a subtask of the more general, and much more ambitious, program of creating a theory of models. (pp. 277-278)

Each of these books, in its own way, sheds new and powerful light on the projects of modeling in general and on theory building about classroom learning in particular.

John Casti: Toward a Science of Complexity

For Casti (1994), systems are collections of elements or agents together with some sort of “binder”–a set of rules or operations that help to delimit the system–that helps to determine just which things are inside and outside of the system. He also points out another piece to the puzzle of studying systems and it seems like a very important subtlety. It is that “no system lives in isolation” (p. 278), that is, any time we look at a system, we always do so *from the context of another system*. This becomes important, because, as the author puts it, “complexity is an inherently subjective concept; what is complex depends on how you look… whatever complexity such systems have is a joint property of the system and it is interaction with another system, most often an observer or a controller” (p. 269). Systems are understood in interaction with other systems. This key aspect of Casti’s systems perspective tells us that complexity is a subjective phenomenon: systems become complex (or not) as a function of the vantage point of the observer.

The term “complex” may be thus ill defined, or at least multiply defined, representing different things for different people and in different contexts. One of the goals of *Complexification* is to begin to “translate some of these informal notions about the complex and the commonplace into a more formal, stylized language” (p. 270). Casti wants to move informal notions of complexity toward “a science” essentially by rendering those notions down to formalisms and relations that can be expressed using the “compact language of mathematics” (p. 3).

A first step in this process can be made by saying what complex systems are *not*. What they are not are simple systems: systems that have predictable behaviors, that involve "a small number of components" and "few interactions and feedback/feedforward loops" (Casti, 1994, p. 271). Simple systems generally have a very limited number of components and they are decomposable–if connections between components are broken the system still functions pretty much as it did before. By counter-example we can begin to see what complex systems *are*: they involve many components (elements, agents), and they are highly interconnected and interactive, involving multiple negative and positive feedback/feedforward loops. They are characterized by decentralized and distributed decision making and they are irreducible – "neglecting any part of the process or severing any of the connections… usually destroys essential aspects of the system's behavior" (p. 272).

Complexity is the foundation of a science of surprise and surprise occurs literally when our expectations and observations are at odds with each other. Casti (1994) prefaces five chapters with what he calls "intuitions," actually, misconceptions, about how the world behaves, that have their genesis in linear and simplistic models, and that routinely lead to unanticipated and surprising results.

Before proceeding, it should be made clear that this entire section on Casti's *Complexification* is intended to inform a developing understanding of classroom learning. Here the class is being viewed as a community of learners and an obviously complex system and the previous paragraph describes nicely why classrooms should be viewed as such–they are *not* simple, as defined above, and they *are* complex. Classrooms are composed of many agents–learners and teachers–and each is a decision-maker whose deciding can affect any fraction of the total population. The elements are highly interconnected and irreducible–removing or adding elements always changes the dynamics and the classroom activity is characterized by multiple feedforward/feedback loops. Having said this about the classroom, let us continue with a brief treatment (Table 1.1) of Casti's "causes" of surprise and try and point to a way in which each may be related to classroom learning.

Table 1.1

Casti's (1994) "Intuitions" and "Surprises," and Classroom Learning.

Intuition	Surprise	In Classroom Learning
#1. Small, gradual changes in causes give small, gradual changes in effects. (p. 43)	Catastrophe theory–small changes in parameters can lead to large discontinuous shifts in related values. This is literally the effect of falling off a cliff. (chap. 2)	Small changes in locus of control from teacher to students may lead to significant changes in classroom learning.
#2. Deterministic rules of behavior give rise to completely predictable events. (p. 85)	Chaos theory – the "Lorenz Butterfly Effect" where minute differences in initial conditions evolve quickly into vastly different states. (Chap. 3)	Regardless of how concrete, straightforward, and simplistic direct instruction may be, learners often emerge with radically different understandings.

#3. All real-world truths are the logical outcome of following a set of rules. (p. 115)	Incomputability – Gödel's Incompleteness Theorem. The essence of this property is that "there's always something out there in the real world that resists being fenced in by a deductive argument" (Chap. 4, p. 150).	Learning outcomes occur in classrooms that current theories of learning are unable to predict.
# 4. Complicated systems can always be understood by breaking them down into simpler parts.	Irreducibility – in complex systems, due to the nature of the connectivity between elements, trying to make sense of the system by breaking the connections irrevocably changes the nature of the system. (Chap. 5)	This is essentially Aristotle's truism that the whole is more than the sum of the parts. Classroom learning is a function of the relations and interrelationships of all the members of the learning community.
# 5. Surprising behavior results only from complicated, hard-to-understand interactions	Emergence and self-organization – "surprising behavior can occur as a consequence of the	Exciting learning outcomes can be achieved by encouraging learners to "self-organize," that is, to

among a system's component parts.	interaction among simple parts" (Chap. 6, p. 230).	direct and make sense of their own learning.

In Casti's "roots of surprise" we have the beginnings of an understanding of complexity as well as the beginnings of a rationale for thinking of classroom learning as a complex system.

To summarize, John Casti (1994) does an excellent job of identifying the challenges in trying to understand experience from a systems-theoretical point of view. Systems that are complex have local instabilities and non-linearities that can result in catastrophic reorganizations, and they exhibit "deterministic chaos," eventually settling in on a few "strange attractors" (p. 29; see Circle-10 System example, pp. 33-37) via chaotic and apparently random paths. Complex systems are ones in which logical rules do not necessarily lead to logical behaviors. They cannot be studied by breaking them into constituent parts because local and global connectivity are critical to the activity of the system. Finally, complex systems are ones in which unexpected patterns at one level can emerge from relatively simple interactions between agents at a lower level.

When we undertake the business of trying to make sense of learning in classrooms and to look at the "big picture"–modeling classrooms as connected wholes rather than linear combinations of individual learners–new kinds of theoretical tools are called for. In *Complexification* Casti has paved the way for the application of complex systems analyses by pointing out that common-sense attempts at understanding systems are usually inadequate, and he has issued a challenge to theory-builders to embrace complexity as a science on their way to the "more ambitious… program of creating a theory of models" (p. 278):

> …common usage of the term *complex* is informal. The word is typically employed as a name for something that seems counterintuitive, unpredictable, or just plain hard to pin down. So if it is a genuine *science* of complex systems we are after and not just anecdotal accounts based on vague personal opinions, we're going to have to translate some of these informal notions about the complex and the commonplace into a more formal, stylized language, one in which intuition and meaning can be more or less faithfully captured in symbols and syntax. (p. 270)

This article will respond to Casti's call for formalization and operationalization of terms and approaches in building complex systems analyses, first by discussing some Camazine, et al.'s (2001) work in the field of biology and then by taking a closer look at John Holland's (1995) book *Hidden Order*.

Camazine, et al.: Biological Systems

The approach to a complexity-based theory of biological organization offered by Camazine, et al., (2001) may be very fruitful for assisting in our thinking about classroom learning in systems sorts of ways. In their book, *Self-Organization in Biological Systems*, these authors describe a wide variety of biological systems and assess the development of those systems in terms of possible "mechanisms of pattern formation" (p. 47) that could be seen as sources of observable organization. When trying to make sense of the complex behaviors of such systems, it is often beneficial to move to a higher-level vantage point and watch the behavioral patterns evolve from the context of a larger set of patterns and behaviors. Alternatively, it is sometimes useful to slip down a level, and observe how the system you are trying to understand offers the context for a "lower" set of behaviors.

For example, if one wants to think about the rules-set that might govern the flight characteristics of one of Conrad Parker's and Craig Reynolds' Java "boids" (http://www.vergenet.net/~conrad/boids/), one can raise his or her vantage point up to the level of the flock. From that position, one can focus one's attention on an individual agent *and* watch the patterns of the flock as a whole, and use this combined perspective to inform attempts at modeling the rules-system of the boid. Similarly, if one is trying to understand the search behaviors of individual foraging ants, he or she could slip "down" a level, and focus on the local terrain and distributions of food sources. It is within this context that the ants' activity patterns emerge–the "structure" of the ants' surroundings exerts a strong influence on observed foraging patterns.

Although Camazine et al. (2001) fail to provide an explicit definition for what they are calling a complex system, it is reasonable to suggest that they are thinking of (agent-based) systems in a fairly standard sense–systems composed of multiple agents acting according to a hypothesized collection of (internal) rules in co-operation with local information and other agents. Activity of the agents at the local level generates patterns at a more global level. These authors define *pattern* as a "particular, organized arrangement of objects in space or time" (p. 8), state that global patterns are seen to emerge from the organization of local activity, and propose various plausible mechanisms for their emergence.

In order to understand the activity of an organizing system, for example, of a classroom as it learns, careful attention should be paid to the types of things that appear to drive or encourage the organization. Camazine et al. (2001) provide a list of potential activity organizers in biological systems: strong leaders, blueprints, recipes, templates, and self-organization. The first four of these impose organization from *outside* the system. The fifth, self-organization, has its genesis *inside* the system: “Pattern formation occurs through interactions internal to the system, without intervention by external directing influences” (p. 7). Beyond successfully responding to Casti’s call for formalization of systems-theoretical approaches, additional “cash value” (James, 1975/1978, p. 32) of Camazine et al.’s systems perspective for the current purposes is obtained from the way that it highlights considerations of control of activity and locates that control internally or externally to the group and its constituents.

Strong leaders, blueprints, recipes, and templates are all mechanisms for controlling activity that are viewed as being external to the system, organizing activity by remote control so to speak. Referring back to Casti (1994), note that systems analyses are “inherently subjective” (p. 269): what one ends up seeing in part depends upon which levels of organization one chooses to observe. In high school classrooms one may view students’ activity as always being driven externally, since, for example, school attendance (on the level of a “long” time frame) is mandatory for most students and periodic class changes (on the level of “short” time frames) powerfully regulate students’ experiences. Mandatory attendance or relocations notwithstanding, even when students are physically present, they usually direct their own attention, deciding (more or less consciously) if, when, and how to engage in learning opportunities and other activities. This ambiguity–are the students being externally controlled or are they directing their own activity–does not invalidate using systems perspectives as ways to study classroom learning. What it does do is demonstrate the need of researchers to be careful about how they delimit, define, and communicate about the systems they are studying.

For example, although it seems that much of what goes on at the level of classrooms in a school *is* actually driven by external controllers such as legislative mandates, “core curricula”, and bus schedules, it also seems like many of the more interesting aspects of learning will only come into focus when we (subjectively) choose to background exterior driving mechanisms and observe classrooms at a finer level of detail. At the level of small groups learning addition facts, whole groups learning about the Civil War, or individuals learning to read, we can begin to look for components of complex systems as defined in other fields of research. As students and groups of students self-organize, selectively negotiating and deciding which chunks of the curriculum to attend to and composing what they have attended to into useful models, it seems that powerful insights into learning (e.g., what is motivating students, what concepts are salient to the students, or how they use the models they have created) will become evident.

Camazine et al. (2001) describe their notion of self-organization as being a function of the concerted activities of groups of self-directed agents within the system. The primary mechanism for the self-organization of groups of agents is *feedback*, another of the important “basic terms” that needs formalization in systems-theoretical perspectives. Positive and negative feedback are “the two basic modes of interaction among the components of self-organizing systems” (p. 15). Positive feedback “generally promotes changes in a system” taking “an initial change in a system and [reinforcing] that change in the *same* direction as the initial deviation” (p. 17). Negative feedback is the mechanism by which the "amplifying nature of positive feedback" (p. 19) is moderated. Negative feedback reacts to changes in the system, triggering an "opposing response that counteracts the perturbation" (p. 16). In these authors' approach to understanding systems, "self-enhancing positive feedback coupled with antagonistic negative feedback [provide] powerful [mechanisms for] creating structure and pattern" (p. 20), keeping the system dynamically dancing along the fine edge between chaos and stability.

In addition to feedback, Camazine et al. (2001) characterize self-organizing systems as being *dynamic*, requiring "continual interactions among lower-level components to produce and maintain structure" (p. 29). Closely related to their dynamism, self-organizing systems are said to be *emergent*, where emergence is seen as a "process by which a system of interacting subunits acquires qualitatively new properties that cannot be understood as the simple addition of their individual contributions" (p. 31). Evolution in time in combination with nonlinear interactions between the agents of the system result in the emergence of complex global patterns that do not exist at the local level.

Camazine et al. (2001) have contributed significantly to the sort of challenge that John Casti (1994) has identified—their seminal work on self-organization has clearly defined terms of interest and types of organization and has helped to clarify their meanings via extensive examples of biological systems. For them, the focus is on self-organization and the mechanisms that bring it about. Following their lead, a researcher would turn his or her attention to the dynamic activity of the agents in a system and look for the mechanisms of positive and negative feedback that cause complex activity patterns to emerge. Camazine et al.'s is an example of a useful and informative, formalized, application of systems-theoretical perspectives in service of understanding the behavior of real-world phenomena. Now let us turn our attention to another noteworthy example of the effort to formalize tools and analytical perspectives on the study of complex systems.

John Holland: Complex Adaptive Systems

Perhaps the most promising and fruitful discussion of complex systems for theory-building about classroom learning is John Holland's 1995 book, *Hidden Order*. In this work, Holland takes on the task of attempting to lay out and define a more-or-less universal and prototypical complex adaptive system. He conjectures that it is possible to identify a set of attributes that all complex adaptive systems (CAS)[3] (pp. 6-10) can be seen to possess. Later in first chapter Holland describes CAS as being primarily characterized by the presence of agents, meta-agents, and adaptation and says that these can be understood and studied in terms of "seven basics," a set of four "properties" (aggregation, nonlinearity, flows, and diversity) and a set three "mechanisms" (tagging, internal models, and building blocks). These features, adaptation, agents (that can be seen as aggregating into meta-agents), and the mechanisms and properties serve two fundamental purposes. First, they can be used in deciding whether or not a system under study can indeed be thought of as a CAS, and second, the features, mechanisms, and properties provide analytical tools for investigating the nature of that system.

First of all, Holland (1995) sets out the notions of "agents" and "meta-agents" as being present in all CAS and requisite for categorizing a system as such. The "active elements" that are "diverse in both form and capability" (p. 6) are seen to act, to behave, as if they were responding to an internal set of rules. He is quick to point out that these hypothesized rule sets are not necessarily the rules that govern the behavior of the agents, or that rule sets are actually what governs the agents' behavior. Instead, *thinking* of the agents as rules-driven in this way provides " a convenient way to describe agent strategies" (p. 8). Next, Holland demonstrates a common strategy employed in systems analyses by bumping up a level and describing "meta-agents" as a way of thinking about "*what CAS do*" (p. 11)–meta-agents are higher-level agents whose complex behavior patterns are actually aggregated combinations of the behaviors of the less complex agents a level down in the hierarchy. In the same way meta-agents can be aggregated into "meta-meta-agents," thus creating "the hierarchical organization so typical of CAS" (p. 9). To summarize, CAS are composed of active elements (in classrooms, these could be individual students or groups of students) that behave as if responding to an internal set of rules, and their behaviors at the local level combine to create informative patterns of activity at subsequent meta-levels. Those meta-levels can again be thought of as (aggregated) agents operating at still higher levels of assimilation

Adaptation, "the sine qua non of CAS" (Holland, 1995, p. 8), is, at least for the purposes of this article, *learning*. Adaptation is a feature of all CAS and Holland often resorts to biological metaphors to characterize this attribute. He says that adaptation is how the "organism fits itself to its environment" and "experience guides changes in the organism's structure… [in order to] make better use of its environment for its own ends" (p. 9). If agents behave as if they were responding to an internal set of rules or a particular internal model, then a way for them to learn is by modifying those rules or that model in response to experience. Rules can thus be viewed as "hypotheses that are undergoing testing and confirmation" (p. 53) and internal models as dynamic representations of the organism's environment. Holland's treatment goes into significant detail about how the transformation of rules and models might proceed, but it is sufficient for the present purpose to say that the process is recursive, based on information (feedback) from the environment that the agent is immersed in and selectively attends to, and results in the formulation of new and improved rules sets.

One other component of Holland's (1995) view of adaptation needs mentioning because it sheds light on the ways in which patterns emerge from the activity of individual agents. Each agent's environment is partly composed of other agents, "so that a portion of any agent's efforts at adaptation is spent adapting to other adaptive agents. This one feature is a major source of the temporal patterns that CAS generate" (p. 10). The agents of a complex adaptive system are constantly adapting to their environment, and that environment includes other adapting agents, the net effect being the evolution of complex patterns of activity when viewed from "one level up". At this point the notion of adaptation is probably sufficiently defined for the purposes of this essay: adaptation can be viewed as learning based upon emergent interactions that an agent has with its environment, a major component of that environment is other adaptive agents, and the patterns of activity generated by agents' individual and mutual adaptation provide the observable organizational characteristics of CAS.

It is important to note, for the purposes of constructing connections to prior theories in the field of educational research (Piaget's in particular), that Holland relies very heavily on "genetic algorithms" (p. 69) and molecular biology to build his theory of the mechanisms of adaptation in CAS. At the same time, lest the readers think that Holland's work is solely biological, it should be noted that he also applies his adaptation schemes to the (iterated) Prisoner's Dilemma as well as economics (pp. 80-87) and many other diverse systems. It is also important to note that as agents and their behaviors evolve, so does their environment. Although Holland does not discuss this explicitly, the idea that agents and their environs can be seen as mutually adaptive provides an additional and potentially useful piece to the puzzle of CAS. The patterns of activity and adaptation of the agents in a CAS are influenced by and influence the environment, and so it seems that watching the evolution of the surroundings as well as the evolution of the agents should also enhance understandings about CAS. This article proceeds with a brief examination of the mechanisms and properties that make up the foundations of John Holland's theory of complex adaptive systems.

The Seven Basics: Mechanisms and Properties

These "seven basics" that Holland (1995) considers common characteristics of all CAS "are not the only basics that could be selected" (p. 10) as ways of understanding the activity of complex adaptive systems. He reminds us that, as researchers, we still need to be a bit artful, choosing which characteristics will provide useful foci for our particular investigations. "This is not so much a matter of correct or incorrect… as it is a matter of what questions are being investigated" (p. 8). At the same time, Holland's work is intended to generate a model that can be used for studying *all* CAS and he claims that "all the other candidates" for mechanisms and properties that he has encountered can be "derived" from "combinations of these seven" (p. 10). This is a strong claim, and one that will need ongoing reflection and testing, but for now, let us accept these seven basics as sufficiently characterizing all CAS and take a closer look at them in hopes of developing a general understanding of their applicability for building useful understandings of complex adaptive systems[4].

Aggregation.

This attribute has two interpretations that are applicable to CAS. First, the simpler sense of this term has to do with the natural process of building categories, and this activity is a fundamental method for building models. It is what we do as model builders. We chose which aspects of a system to aggregate in order to simplify the complexity–this is one of those "artful" activities that helps us to make sense of the world. So in studying a system, or thinking about our worlds, we very naturally create inclusive categories such as cars or trucks or "gifted" learners. A subtlety that figures later into the conversation about "building blocks" is the idea that the categories we create are "*reusable* [emphasis added]; we almost always decompose novel scenes into familiar categories" (Holland, 1995, pp. 10-11). Aggregation in this (simpler) sense speaks to the categorization of components of CAS that are chosen for the purposes of highlighting certain features and backgrounding others in service of a particular investigation or model.

The second sense of the term aggregate is the one mentioned in the foregoing introduction to Holland's approach. This sense of the term refers to the coalescence of individual agents at one level of complexity into "meta-agents" at the next higher organizational level, and is a fundamental property of all CAS. Careful study of this type of aggregation is one of the primary means by which we can make sense of these systems and this complex systems approach. Holland poses several questions germane to this type of aggregation:

> What kind of "boundaries" demarcate these adaptive aggregates? How are the agent interactions within these boundaries directed and coordinated? How do the contained interactions generate behaviors that transcend the behaviors of the component elements? We must be able to answer such questions if we are to resolve the mysteries… (p. 12)

These questions point at means by which we can begin to use elements of CAS analysis for the purposes of furthering understanding of particular systems. In the case of classroom learning for instance, what will be the composition of spontaneously forming "small groups" in a given classroom, and how will those aggregates evolve over different time scales? On what basis, what sets of rules, will these groups form? What are the effects of these small group formations on learning at the individual, small group and whole class levels? One possible means for investigating these questions is the first item on Holland's list of seven basics–tagging–and it is to this mechanism that our attention will now turn.

Tagging.

This is one of the *mechanisms* of CAS that enables adaptation and promotes the formation of aggregates. Tagging is a process of identifying features in the environment of a CAS that become salient and useful in determining its future activity. A CAS selects salient features (building blocks) from all the possible inputs in its environment as a function of a currently active set of tagging rules and these rules structure agents' parsing their environments by motivating and driving selective attention. When a CAS first encounters a situation, a preexisting set of tagging rules relevant to the particular situation becomes active, and the rules specify particular things for the CAS to expect, and to look for. “Well-established tag-based interactions provide a sound basis for filtering, specialization, and cooperation” (pp. 14-15), and these activities in turn lead “to the emergence of meta-agents and organizations” (p. 15). It is important to note that tagging rules sets are themselves persistent internal models (“schemata,” p. 90) that are composed of building blocks derived from useful tagging strategies and model building in earlier experiences. The notion of “tagging” as a mechanism for aggregation and adaptation is a powerful affordance of systems-theoretic approaches for understanding classroom learning, and it can help provide useful answers to the lists of education- and learning-based questions posed above.

Nonlinearity.

Holland’s (1995) definition of this *property* of all CAS is equivalent to its common usage in mathematics. Simply put, it states that the behavior of the whole cannot be understood by a simple additive combination of the parts. This important property is another of the things that make CAS *complex*. Multiple considerations figure into the activities of the agents in a system, and depending on local spatial and temporal conditions various “weight functions” associated with various rules combine to produce decisions of the moment. As it turns out, relatively few of the truly interesting things in life can be accurately mathematized using strictly linear functions–as Holland puts it, “To attempt to study cas with [linear] techniques is much like trying to play chess by collecting statistics on the way pieces move” (pp. 15-16). Complex systems simply cannot be accurately described using linear mathematics. One important implication of using a complex systems approach to understanding classroom learning, and learning in general, is just this point. That is, a complexity-based view of learning will be focused on the non-linear properties of the system, and their presence makes the use of simple averaging techniques (e.g., "bell curves") unreliable and outright inadmissible methods for complex systems analyses. Current efforts to measure the effectiveness of teachers by looking at the aggregated scores of their students on high-stakes assessments are questionable in light of the non-linearities inherent in the CAS called classroom learning.

Flows.

Understanding of this *property* of complex adaptive systems is facilitated by thinking of CAS as networks of nodes and connectors. The nodes might be small towns and local roads the connectors that enable the flows of goods and services. Tags play an important part in the development and evolution of flows–because the "adaptive processes that modify CAS [flows] select for tags that mediate useful interactions and against tags that cause malfunctions" (Holland, 1995, p. 23). Flows are observables that evolve, coming and going in space and time and as such they provide insights into the workings of a CAS as it adapts. There are two important properties of flows, multiplier and recycler effects. The multiplier effect relates to the situation where resources are "injected" at a node, and the recycler effect speaks to the situation where the "stuff" of flows is returned to the network (p. 23). Both of these effects are in the category of "positive feedback" and are potent sources of nonlinearities. Examples of things that "flow" as classrooms learn might be information, control over time, or material and equipment resource allocations. Flows are another property that can provide information about and enable developing understandings of complex adaptive systems.

Diversity.

Another *property* of Holland's CAS, diversity plays a fairly complicated role in their makeup. To understand diversity, one needs to think about the "niches" that agents may fill, in, for example, a biological ecology and to think about how those niches might evolve over time. "Each kind of agent fills a niche that is defined by the interactions centering on that agent. If we remove one kind of agent from the system, creating a 'hole', the system typically responds with a cascade of adaptations resulting in a new agent that 'fills the hole'" (Holland, 1995, p. 27). The property of diversity is a major factor in the evolution of an ecology when, for example, an agent moves into totally new territory or when an agent is successful in generating a new niche. Diversity is a "dynamic pattern, often persistent and coherent like a standing wave" (p. 29), but it is actually more dynamic than a standing wave, because the diversity property itself evolves as a function of adaptations, opening the "possibility for further interactions and new niches" (p. 29).

Here we can really see the method in Holland's ordering of these properties and mechanisms–diversity depends in a non-linear way on flows, and in return, as niches open and close, diversity alters the flows in the CAS. Tagging is a major determinant of *what* flows and *how* it flows. Evolution of flows occurs in very non-linear ways, and the whole process constitutes and is constituted by adaptation. Perhaps the readers will agree that what is being built here is a cogent, cohesive, and inclusive model for thinking about, talking about, and investigating the nature and evolution of complex systems such as classroom communities of learners.

Internal Models.

The *mechanism* of internal models plays a vital role in the activities of CAS. Internal models are the mechanism by which CAS anticipate, and it is through anticipation and prediction that agents adapt to and thrive in their environments. Although the mechanism of internal model building seems much more applicable to sentient systems than to *all* CAS, Holland (1995) makes the case that even bacteria implicitly predict the presence of food (i.e., build an internal model) when they follow chemical gradients (p. 32). It is important for Holland's work in developing a universal model of CAS that he be able to identify a way that prediction and anticipation work at (essentially) all levels of CAS analysis (thus including lower life forms), but educators and educational researchers do not share that constraint. In human learning in general, and in classroom learning in particular, the notions of anticipation and prediction based on internal models are not at all difficult to defend.

According to Holland (1995), the "critical characteristic" of a model is that it enables the agent to "infer something about the thing being modeled" (p. 33). Internal models are created by an agent's selectively attending to building blocks in its environment and then using this information for the purposes of creating and refining its internal structure, its models. The models are then employed as predictors, elements internal to the agent that enable it to respond to and benefit from the local environment. Models "actively determine the agent's behavior" (p. 34). They are "subject to selection and progressive adaptation" based on new information, and we start to see the possibility of iterative adaptational loops, based solely on individual agents and local conditions, that can provide powerful insights into the learning and adaptation patterns of higher-level

CAS (meta-agents). In a classroom example, learners may create and refine their internal knowledge structures as a function of interactions with their environment, and at the same time, the meta-agent, the classroom, may change its nature in a related manner.

Building Blocks.

The last of Holland's seven basic ingredients of complex adaptive systems is closely related to the mechanism of internal models. This *mechanism* provides a means for generating useful internal models of a "perpetually novel environment" (Holland, 1995, p. 34) by the distillation from experience of reusable "building blocks." Agents create building blocks through a process of selective attention–decomposing information from their environment into constituent elements that can be combined and re-combined into internal models. Through iterative use and testing agents accumulate building blocks that enable the construction of useful internal models, models that enable those agents to anticipate the probabilities and consequences of potential actions. Iterated and mutually influential development of building blocks and internal models is a key source of the adaptation of complex adaptive systems.

Summary

John Holland's (1995) treatment of the fundamentals of complex adaptive systems is a detailed and comprehensive systems-theoretical framework. It actually provides a well-defined "litmus test" for deciding whether or not to consider a system as complex and adaptive and for defending such a decision. Beyond that, it provides powerful conceptual tools, the properties and mechanisms, for analyzing the activity and pattern development of CAS.

This extended discussion of Holland's model of generalized complex adaptive systems is intended to provide a subsequent framework for building arguments that classroom learning and human learning in general can be profitably studied from a systems-theoretical point of view. That project might proceed by first drawing parallels between observed patterns of behaviors, mechanisms, and properties in classrooms, and then using those parallels to generate a model of learning based on the systems approach. Although a thorough treatment of classroom learning as CAS is beyond the intended scope of this article, a plan will be sketched out for how one might proceed toward doing that in a later section.

Each of these three perspectives on dynamical systems, Camazine, et al.'s (2001), Casti's (1994), and Holland's (1995), provides an ontological pathway for making sense of complex systems and of the collective behavior of aggregates of agents. These perspectives also serve to define and clarify the types of considerations necessary in undertaking a complex systems analysis. The next section of this article will turn to a discussion of the utility of these perspectives for understanding the complicated activities inherent in classroom learning.

Individualized and Systems Approaches to Learning

The great majority of theory building in constructivist and cognitivist learning theories has been focused on the learning of an individual. Behaviorist perspectives (Skinner, 1954; Stein, Silbert, & Carnine, 1997), information processing perspectives (Anderson, 1983; Anderson, Reder, & Simon, 2000; Mayer, 1996), novice-expert perspectives (Chi, Feltovich, & Glaser, 1981; NRC, 1999, chap. 2; Reiner, Slotta, Chi, & Resnick, 2000), schema-theoretic perspectives (Derry, 1996; diSessa, 1993), and constructivist perspectives (Cobb, 1994; Ernest, 1996; Piaget[5], 1923/1959, 1924/1969,1929/1951; Vygotsky, 1987, chap. 6) are all predominantly individualistic views of learning. These efforts have provided many insights and have been very successful in helping researchers to build useful models of learning. Although individualistic approaches to learning have been quite productive they also have several limitations.

Limitations of Individualistic Theories of Learning

The first limitation of individualistic theories of learning is that they do not "scale" well–that is, the learning of a classroom of students is not very profitably described as the linear combination of a number of individual learners. This type of scaling to whole classrooms of learners does not and cannot take into account the complex interactions and synergetic effects derived from the properties of groups. Very little of what goes on in classrooms can be understood in terms of straightforward cause and effect relationships and simple aggregations of individual learners.

Individualized models of learning tend to be more static than dynamic. Behavioristic models (e.g., Stein, Silbert, & Carnine, 1997, pp. 3-29) assume that learning is the simple accumulation of fixed and appropriately sized knowledge bits that are taken in as given, without any active adaptation or interpretation on the part of the learner. In another line of (individualized) learning research, learners are posited to possess relatively static conceptual structures and then teaching and learning are thought of as constructing knowledge structures and then repairing or replacing "misconceptions" (Reiner, Slotta, Chi, & Resnick, 2000, p. 7) with increasingly "expert" structures, though little is said in the literature about how these transformations actually take place. Even

Andrea diSessa's schemas of phenomenological primitives (diSessa, 1993, p. 111) may be viewed as static structures that learners access information from for the purposes of making sense of the world around them. In each of these lines of research, knowledge is envisioned as bits of information stored in and accessed from static conceptual structures internal to individual "knowers".

Individualized approaches also tend to focus on a learner at the expense of the learner's context and her or his membership in a learning community. There are many aspects of the surrounding contextual situation that influence how learning takes place and what gets learned (Lave, 1988; Lave &Wenger, 1991; Wertsch, del Rio, & Alvarez, 1995). Students and teachers are embedded in a wide variety of social, historical, and cultural systems (cf., Bowers, Cobb, & McClain, 1999; Hiebert, et al., 1996, p. 19; Lave & Wenger, 1991, pp. 67-69) that profoundly affect learning (Cobb, Perlwitz, & Underwood, 1996) and individualized approaches to learning generally overlook these important and complex influences. The sociocultural historical milieu of the classroom can be seen as the environment relative to which adaptation occurs. At any given moment, classrooms and learners are immersed in a wide variety of interconnected and often competing activities and goals structures. Theories of learning that focus on individuals generally do not take these kinds of complex and ubiquitous learning conditions into account.

Finally, individualized accounts of learning do not offer very much to teachers in the way of helping them to make sense of or design for whole-classroom activities. Although teachers may develop individualized educational plans, they almost never design classroom activities with a single individual in mind. Classroom activity is inherently *group activity*, and there is very little in the language and ideas of individualized cognitively-based or even constructivism-based learning theory that enables teachers to make sense of the activities of groups of learners.

Affordances of Systems-theoretic Approaches to Learning

In contrast to individualized approaches, dynamical systems-theoretical perspectives have much to offer in terms of helping teachers, researchers, and others to focus on and make sense of learning at the level of the group. First, a systems perspective enables thinking about classroom learning in terms of a dynamic, continuously changing "dance" between the group, its members, and the contextual situation. Second, as discussed above, classrooms are much more than a linear sum of individual learners, and a systems perspective enables thinking about the synergetic affordances and "lever points" (Holland, 1995, p. 39) inherent in classrooms. Third, it may well be that the most important affordance of systems-theoretical approaches to learning is in the *language* of complexity itself.

Very recently, complex systems terminology and the ideas that that terminology represents have increasingly been finding their way into the discussions and literature of cognitivist, constructivist, and sociocultural learning theory camps. It seems that the use of the language of complexity is preceding more rigorous and careful application of systems-theoretical tenets–Casti's (1994) "formalization" (pp. 274-276)–to perspectives on learning. The hope is that use of the language will serve as precursor and enabler of more systematic complexity-based modeling in future education research. In any event, the combination of the fundamental ability to address dynamism and complex interrelations, the ability to provide a powerfully descriptive language for talking about what happens in classrooms and larger contexts, and the potential for formalization of these things into useful models all serve to demonstrate the potential of systems-theoretical approaches for research into learning.

Systems perspectives *do* scale well in terms of considerations of group activity. In one way or another, every dynamical systems viewpoint addresses both the individual and the aggregate. For example, from the structuralist perspective of Jean Piaget[6] (1968/1970, chap. 2), the group and the elements of the group are mutually constitutive. That is, the dynamic creation of a group, the activity of individual members of the group, and the group's context each influence the other, forming a complex adaptive system. An example of this in a classroom is when students are aware (or quickly become aware) of their status within the larger group, and those status considerations have powerful effects on the students' and the group's subsequent activities (cf. Empson, 2003). Complex systems analyses (Casti, 1994; Camazine, et al., 2001; Clark, 1997; Holland, 1995, 1998; Stroup & Wilensky, 2000; Prigogine, 1984, 1997) focus on higher-level patterns (e.g., aggregation, flows) that are generated by activity and adaptation at the level of individuals whose behaviors are based solely on the local environment and the individual's own internal models. In contrast to individualized theories of learning, systems-theoretical points of view are fundamentally concerned with viewing learners and groups as mutually constitutive agents whose behavior is to be understood in the context of their larger patterns of activity.

Systems-theoretical points of view tend to be very dynamic–characterizing activity in terms of evolving *patterns* as opposed to looking at activity captured in static "snapshots". A clear-cut and visual example of this can be seen in Conrad Parker and

Craig Reynolds' model of the flocking behavior of birds (http://heidi.vergenet.net/~conrad/boids/). Here the observer is virtually compelled to build an understanding of the "boids'" that is fundamentally dynamic. The back-and-forth, up-and-down, landing-and-taking-flight behavior of real birds is quintessentially captured by the dynamical nature of the modeling. The same will be true of dynamical representations of classroom learning. It will be the *patterns* of activity that become the focus of understandings that develop. Rather than static "snapshots" of individual students' learning, such as quiz grades or end-of-year tests, assessments of learning will be made in real-time, and in context. For example, in the NetLogo (Wilensky, 1999) HubNet 'Function Activity', students' moving toward or away from the targeted locus of points on a line provide learners and instructors with immediate feedback. Learners and teachers can see, assess, and adapt moment by moment—thus understanding the current states of the learners' and learner's progress and proactively modifying those states become integral aspects of ongoing classroom activity. The dynamic character of a systems-theoretic view of learning provides grounding, tools, and a framework for thinking about the patterns of evolution, development, and adaptation of learners and learning.

Complexity and complex interactions are what characterize classrooms and classroom learning. Students' goals and teachers' goals are frequently different and often at odds. Classroom participants' social, economic, cultural, and historical backgrounds are becoming increasingly diverse and sometimes adversarial (Fordham & Ogbu, 1986; Ogbu, 1990). Classroom structures may require behavior that is antithetical to expected behaviors in students' non-school lives. Classroom learning is situated and directed by the contexts of the school, the district, and local, state, and federal mandates, norms, and expectations. All of these factors and many others combine to create a dazzlingly complex *context* for situating learning, and systems-theoretical approaches provide unique advantages for dealing with complex contexts.

Systems theoretical approaches can "see" complex relations and accommodate their effects. The possibility of multiple "attractors" (Casti, 1994, pp. 28-29) within a limited region is taken as a given in CAS approaches. As an example from the classroom, consider the conflicting student goals of wanting to perform well on a test and not wanting to upstage one's peers, or for a teacher, wanting all of one's students to pass first year algebra but not feeling capable of handling large class sizes in second year algebra. Systems approaches attend to the existence of multiple driving forces and have mechanisms for dealing with the concomitant positive and negative feedback effects on the behaviors of the system.

Systems perspectives encourage a reflexive and reflective shifting of levels, from considerations of the individual's perspective to a focus on the patterns of the aggregate and back again. Individuals make personal decisions based upon their own "rules sets" and their immediate situation. The synergetic sum of individuals' decisions gives rise to the patterning of the activity at the group level. A further shift in levels, say from the classroom to the school, might facilitate decision making about curricula and policies. A systems perspective enables multiple levels of focus for the purposes of understanding and making sense of learning.

In addition to the above, systems-theoretical approaches offer a subtler and perhaps much more important benefit. They give researchers, teachers, and learners a consistent language with which to *talk about* classroom learning. The terminology of complexity theory is finding its way into educational discourse at nearly every level. From state-of-the-art researchers to curriculum designers to classroom teachers people are beginning to employ systems ideas in many arenas. This appropriation of the language and the common-sense ideas of complexity by inquirers into learning may be the most compelling argument of all for the utility of the approach. Thinking about groups of learners and the consequences of nonlinearities in learning becomes much more likely and productive as educators appropriate the language of complexity.

FUTURE RESEARCH DIRECTIONS

The implications of systems-theoretical perspectives for research into learning remain largely unexplored, and at best what follows are but a few of the possibilities suggested by the foregoing discussions of the work of John Casti (1994), Camazine, et al. (2001), and John Holland (1995).

From Casti's (1994) perspective, there is a research opportunity in terms of arguing for a definitive demonstration that classrooms and classroom learning can indeed be thought of as "complex" (p. 269) systems. Although the idea of classrooms as being complex systems is apparently being taken as a given by the education research community at large, an effort to identify the characteristics that Casti describes as the "stuff of complexity," such as catastrophic changes, irreducibility, and emergence in real-life classroom activity would be very important. It seems that one of the first steps toward Casti's sort of formalization of a systems approach to classroom learning should be a careful characterization of the range and types of complexification that occur in classrooms. Identification and exemplification of Casti's five sources of surprise (chaps.

2-6) as portrayed in actual classroom videotape footage accompanied by careful analysis of the nonlinearities and the sources thereof would constitute a valuable contribution to research on learning.

The systems perspective developed by Camazine, et al. (2001) offers another possibility for research directed at classroom learning. These authors offer five possible mechanisms of control of the activities of organized systems: strong leaders, blueprints, recipes, templates, and self-organization. The idea is that the activity of aggregates of agents (systems) can be directed *externally* (through the first four mechanisms listed) or *internally* through the process of self-organization, in which the activities of individual agents acting on their own accord based on local information combine to form complex patterns and structures (e.g., termite mounds and beehives). In relation to classroom learning, one can view any of these five "organizers" as the driving force behind particular classroom episodes. The line of possible research might be descriptive, much like the above proposal based on Casti's work, trying to identify and capture episodes of activity where each of the five organizers is the dominant source of patterns of activity. Beyond this type of descriptive analysis, and once substantial patterns of organization and organizers have been established, research into the relative value of each for given learning outcomes might be pursued. For example, it may be that direction from a "strong leader" would prove to be most productive for acquisition of behaviorist-type learning goals such as rote memorization while self-organized learning opportunities might be shown to be more effective for developing higher-level thinking and problem-solving capabilities.

The perspective that seems to have the most potential for development of a systems-theoretical model of learning is John Holland's (1995). Holland has tried to develop a model of a "universal" complex adaptive system. It is his contention that every CAS is composed of agents that aggregate into meta-agents and "learn" by adaptation. Beyond these fundamental attributes, he sees three mechanisms (tagging, building blocks, and internal models) and four properties (non-linearity, diversity, flows, and aggregation) as delineating a framework from which to make sense of CAS. Holland's careful analysis and informative examples provide potentially powerful grounding for the projects of: 1) *describing* classroom learning and individual learning as CAS and, much like the proposal above based on Casti's work, ultimately *defining* such learning as complex; and possibly, 2) extending and using such an analysis in order to *explain*, *predict*, *orchestrate*, and *assess* learning in classroom situations.

Certainly each of these suggestions for research is tentative. The point here has been to begin a discussion and to lay out some ways that systems perspectives might be used for investigations into learning at several scales from individuals to groups. It is safe to say that human learning is not well described by simple models, linear relations, and static snapshots. This review hopefully represents the beginnings of an extensive and fruitful investigation into learning based on systems theories, self-organization, and, as Casti (1994) puts it, a "science of surprise".

CONCLUSION

Since before recorded history humankind has been trying to make sense of a very complicated and complex world. We seek patterns that serve to simplify and distill experience and enable us to deal with complexity and survive, even flourish. Simplistic modeling, approximation with linear relations, and dissection of big problems into small, solvable sub-problems have enabled significant progress and provided good “first order” results. Getting at the second, third, and higher order solutions requires accounting for more complex interrelationships and fabrication of subtler models. The evolution of systems-theoretical perspectives and modeling has paralleled and supported our quest for increasingly fine-tuned understandings. As the mathematics and the tools for computation have become more and more powerful, so have our models. Aristotle saw that the whole was usually greater than the sum of its parts, but it has taken quite a while to develop that observation into something more, something that we could use. From Piaget’s (1968/1970) notions of an algebraic group, to Bertalanffy and general systems theories, to self-organizing biological systems and a generalized “complex adaptive system” (Holland, 1995), we have made a tremendous amount of progress in recent times. We may be witnessing a large-scale paradigmatic shift (Kuhn, 1962) in our worldview where systems perspectives that embrace complexity will supplant linear perspectives that do not.

Bertalanffy (1968) and Forrester (1968) established a general theory of systems. John Casti (1994) compellingly demonstrates that surprising results can emerge from even very simple systems and that our old truisms likely will not be adequate for building higher order models for understanding. Camazine et al. (2001) are deeply involved in building just the sort of “science of surprise” that Casti is lobbying for by closely considering how global patterns of organization emerge. John Holland (1995) is extending the complexity science still further building a "big idea" that captures the general features of all complex adaptive systems and gives us tools for making sense of them. As systems theories become more evolved and more robust, it is only natural that their range of application will increase.

The time has arrived, and the tools are at hand, for educational researchers to begin to build models that embrace the complexity of learning in ways that have not been possible before. We are prepared to move beyond models of learning that reduce it to the pairing of stimulus with response or to a static collection of "data bits" and snapshots of student learning. Systems-theoretical perspectives on learning are providing us with the tools to make that move. The first steps will be necessarily small—we are just beginning to *talk* amongst ourselves about "emergent learning" and "synergetic effects" in groups of learners, but the movement is growing and applications are being honed. As the common use of the language of complexity moves toward formalism, the model building will become more powerful. That is really what writing this article has been about–an attempt to push understandings beyond vague ideas and imprecise meanings toward something approaching, if only roughly, a formalism.

END NOTES

[1] URLs for these facilities can be found in the References section.

[2] Here, "classroom learning" is taken as an emergent conception whose meaning is evolving in relation to developing understandings of learning, systems, and real-life classroom experiences. Rather than leaving such an important term undefined, and with the foregoing caveat, let me say that I think of "classroom learning" in much the same way that I understand Lave and Wenger's (1991) "communities of… practice" (p. 29) combined with moving toward "full participation" (pp. 36-37). That is, I view classroom

learning as a community of learners approaching full participation in larger communities, such as the science or mathematics disciplinary "communities."

[3] Although Dr. Holland uses the lowercase 'cas' as an acronym for complex adaptive systems, this paper uses the uppercase CAS throughout.

[4] Holland discusses the seven basics not according to whether they are mechanisms or properties of CAS, but rather in a manner that emphasizes their interrelationships, and that order will be maintained here, for the same reason.

[5] Although there is controversy over whether or not Piaget was actually an individual-constructivist most of the work done in the Piagetian tradition is decidedly focused on individuals.

[6] Here, we are considering the systems-theoretical, non-individualistic aspect of Piagetian constructivism.

SECTION 2

ORDER IN CHAOS: SCALE-INVARIANT BEHAVIORS IN PARTICIPATORY SIMULATIONS

Casual observations of the activity of students engaged in participatory simulations (Wilensky & Stroup, 1999) appear quite chaotic—two-dozen people all pushing buttons and making moves more or less simultaneously and at will. In the midst of this apparent chaos, there is a measure of the group's simulation activity that is remarkably organized. A trademark of complex systems is that they experience apparently chaotic fluctuations in the course of their evolution. We say apparently chaotic because those same fluctuations, when viewed from a different perspective demonstrate a fairly startling regularity. In previous work, Thomas Hills and co-researchers (Hills, Hurford, Stroup, & Lesh, 2007) found scale-invariant behaviors in students' participation in complex-systems simulations and discussed how particular patterns of behavior correlated with success in the simulation. We found Zipf-like (Zipf, 1949) scale-invariant behaviors (Adamic, n.d.) in the activity of individuals, classrooms, and aggregations of classrooms.

The current article reports on replication and extension of the previous work and concludes by demonstrating that the same scale-invariant behaviors can be found in three different types of "participatory simulations" (Wilensky & Stroup, 1999). Furthermore, argue for notions of optimization as potentially informative ways of thinking about the emergence of scale-invariance in learning situations.

Design-based research (DBR) in education and the learning sciences has two fundamental goals—development of theoretical knowledge and generation of educative learning activities. The data provided here were obtained in an early iteration of a DBR-program that aims to build theory related to *learning as a complex system* and to design effective learning *environments for learning and teaching about complex systems*. A report on the activity-development component of the current work can be found elsewhere (Hurford, in preparation). In the present article, we report on the theory-building component of our design research experiment. We assume that classroom activity in general and learning in particular and can be viewed as complex systems and we report empirical data that supports our assumption.

Complexity and Learning

Complex systems are ubiquitous in many fields of study and multiple disciplines have benefited from the adoption of complexity-based models, metaphors, and theories. While "complex systems" predictably, means different things in different fields, there are some characteristics that can be reasonably expected in most complexity approaches. First, there should be multiple agents, dynamic activity, aggregation, and adaptation. The agents should be autonomous, and their activity should be "self-organized" (Camazine, et al., 2003, p. 8). That is, patterns of behavior observed at a level up (Casti, 1994, pp. 17-19) from the agents should emerge from the autonomous decisions of individual agents, and each agent's decisions should be made solely on the basis of its local environment. Complex systems tend to be sensitive to initial conditions, experience phase transitions, and exhibit scale-invariance in some aspects of their (dynamic) behavior. Models and understandings generated in various complexity fields provide much of the fundamental theoretical underpinnings of this research.

Complex adaptive systems (CAS) are complex systems as above that also have the ability to adapt. John Holland has developed a model of CAS in which he says that adaptation is how the "organism fits itself to its environment," and that "experience guides changes in the organism's structure… [in order to] make better use of its environment for its own ends" (Holland, 1995, p. 9). Agents acting in relation to their surroundings do so in order to be more successful in that local environment. Activity at the local level is the source of emergent patterns viewed from one or more levels up. So, for example, the day-to-day activities of individual bees, all performed relative to each bee's immediate surroundings, results in the very organized and self-sustaining pattern that we call beehives.

Learning *as* a complex adaptive system (LCAS), the idea we are trying to develop, is a model of classroom learning that is closely patterned after John Holland's CAS (1995). While our larger research agenda includes several levels of modeling learning as a complex system, this portion of the project is solely focused on the search for scale-invariant behaviors across several levels (individuals, classrooms of students, and aggregations of classrooms) of organization. Finding scale-invariance in classroom settings is a powerful indicator that these environments, and the activity that occurs in them, can be profitably thought of as (highly adaptive) complex systems.

Scale invariance is a property of complex systems that is the result of chaotic behaviors at phase transitions (c.f., Cancho & Sole´, 2003). The behaviors of agents self-organize and produce stable patterns of activity when viewed from another system—that is, from some number of levels up. The patterns that emerge produce power-law relations between selected system parameters in contrast to the more familiar Gaussian, bell-shaped types of patterns. In Gaussian distributions, it is easy to find a central value that best approximates the distribution as a whole, and variations that are much different from average are highly unlikely. In power-law relations, there is no notion of an average value, and instead there is a non-zero probability of a broad spectrum of values across, in some cases, many levels of magnitude. For the sake of clarity we mention that we consider scale-invariance and self-similarity to be different ways of describing the same phenomenon.

This research project has required and will continue to require the development of a collection of conceptual and computer-based tools. Complexity-based models of learning both structure the research and are the aims of the research. We view learning as a complex adaptive system and that helps us to determine what to look for and what to expect. As we gain experience doing that, we intend to revise and improve our conceptual model so that it can be more useful to researchers, and will become useful for practitioners, and ultimately, learners. We are also developing conceptual and computer-based tools for designing and building classroom activities (participatory simulations) that can simultaneously provide us with extensive databases on how people actually engaged with our learning activities.

The results of this iteration of design-based research replicate previous findings, namely, that subjects engaged in participatory simulations exhibit scale-invariant behaviors. Beyond replication, the current work extends the previous work by identifying the same type of scale-invariance in two other participatory simulations. We also provide a brief discussion of some of the postulated sources of scale-invariance and a tentative outline of an alternative approach relative to learning that focuses on learner optimization processes. We conclude by offering a brief discussion of future work.

Methods

We collected data from a convenience sample of three pre-service teacher education classrooms participating in a simple whole-class algebra activity, a biology-related foraging simulation (Hills & Stroup, 2004), and a disease simulation (Wilensky & Stroup, 2002), all built in the HubNet participatory simulations environment (Wilensky & Stroup, 1999). Each of these activities included participants (pre-service mathematics and science teachers) using laptop computers that were wirelessly connected to a central computer. The central computer hosted the simulation and projected the simulation interface for each of the activities onto a large screen at the front of the classroom. Students were represented in the up front space by a variety of avatars so that they could find themselves and see where they were in relation to other participants and the field. The participants' computers had interfaces with buttons to push for moving in the field and that provided them with important information about the location, direction of motion, amounts of resources they had acquired, and so on.

Extremely powerful affordances of the HubNet/NetLogo software environment include the types and quantities of information that the software enables us to collect. The "export world" feature of NetLogo allows us to capture the entire state of the system across time frames as small as tenths of seconds and store the information in data files. We can then re-play the simulation in its entirety and extract any variable quantities (e.g., times between moves, sick or well, location and direction of avatar, etcetera) for any or all of the participants. In each of the simulation-experiments that we report on here, we were solely concerned with the time-between-moves of the participants in the HubNet activities.

Algebra Activity

This is a very simple activity that we often use to help new participants get familiar with HubNet, with the interfaces on their computers and with the up-front space.

To begin, we started once more with a 31 by 31 grid of patches and the software placed the participant-avatars at random locations on the grid. The interface on each individual's computer registered the x- and y-coordinates of their current position and they were able to move up, down, left, or right one patch at a time. The students were instructed to "move until your y-value is the same as your x-value," and as they did a diagonal line of avatars emerged in the up-front space. Again the technology enables us to capture all pertinent information about participants' movements and elapsed time between moves.

Forager Simulation

In the foraging simulation, each participant controlled the search patterns of an individual forager-avatar in the 2-dimensional field of the up-front space. The participants' goal was to find invisible "food" patches that were grouped together in small regularly shaped clumps (Figure 2.1). The field was a 31 by 31 square grid of patches that might or might not contain resources. Participants could move their avatars up, down, left, or right one patch at a time for each individual move and the simulation speed was such that avatars responded in less than 250ms. As participants moved around in this simulation, they were not able to see the locations of the food resources but could see themselves and their own locations and the movements of the other players in the upfront space. Monitors on the HubNet interface informed players as to whether or not they were on a patch with food and their total accumulation of food resources. Figure 2.1 shows food patches in orange for illustration purposes, but the participants could not see them. Counting monitors on individuals' interfaces provided participants with their only indications that they were on a food source.

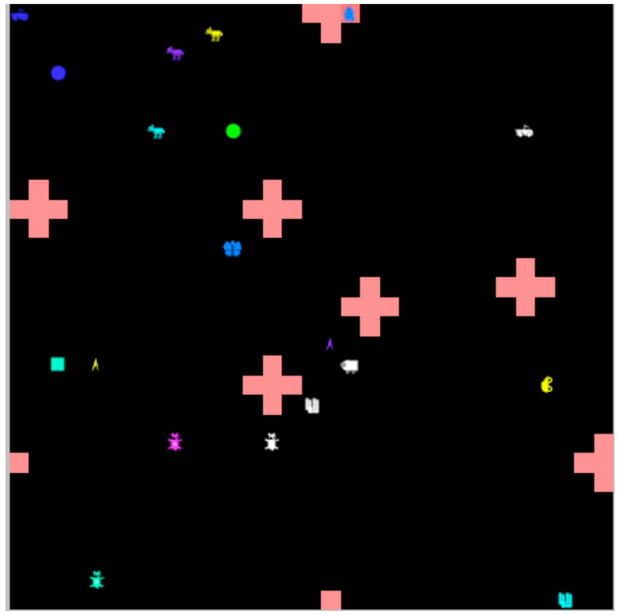


Figure 2.1 A screen-shot of the Forager playing field. The pink crosses are food resources, the other icons on the screen are participants in the simulation.

By moving around the playing field participants could construct mental images of the underlying distribution of resources and as they encountered patches with food the computer interface registered the "hit." Using this information, they could then adapt their foraging behavior to take advantage of the fact that the food was distributed in clumps, working carefully around the nearby patches. Students could also listen to and see the actions of the other participants and use that information to make decisions about where to move next. Separate resources were allocated for each participant, so students were not competing for resources and resource locations were the same for all players. Throughout the simulation we recorded all information about the avatars including position, resources acquired, and wait times between moves in the simulation space.

Disease Participatory Simulation

In this simulation of the way a disease might propagate through a population we begin with a square grid appropriately sized to accommodate the number of participants, in this case, with a 21 by 21 patches grid. The participants are able to move up, down, left, or right and are able to choose their “step size” to be integer numbers of patches from one to five. The participants’ primary goal in this simulation is to evade contagious avatars and remain well for as long as possible. The computer interfaces have the usual motion buttons and a monitor box that indicates if they are infected or not. The up-front space displays the positions and motions of each of the participant-avatars, whether an avatar is infected, and a graph that displays the number of infected participants for every tick of the clock. In addition, the simulation marks avatars with a highly visible red dot when they are infected and has the ability add “androids,” additional avatars that that move randomly and can become ill and infect others. The software enables us to capture every simulation variable at every tick of the clock, thus giving us the capability to actually re-play the simulation at a later time.

Data Analysis

For the purposes of this experiment we were only interested in the time that participants took between “button-presses,” that is, in the elapsed time between participants’ moves in the up-front space and we captured that information for every participant in every run of the simulations. We measured the time participants took between button-presses in the simulation and, following Zipf (1949), calculated the log of individual decision times. Those data are plotted against the log of the rank of the individual decision times where the rank is simply a rank ordering of decision times by their duration. The data were analyzed at the level of individuals, classes, and an aggregation of all participants. The data were plotted, least-squares lines were fitted to the data and correlation constants were calculated. At this early juncture in the research we are not able to assign meaning to the slopes of the curves or their x- or y-intercepts and are only concerned with the high degrees of linearity in the relationships between logs of the decision times and the logs of the ranks of the decision times.

Results

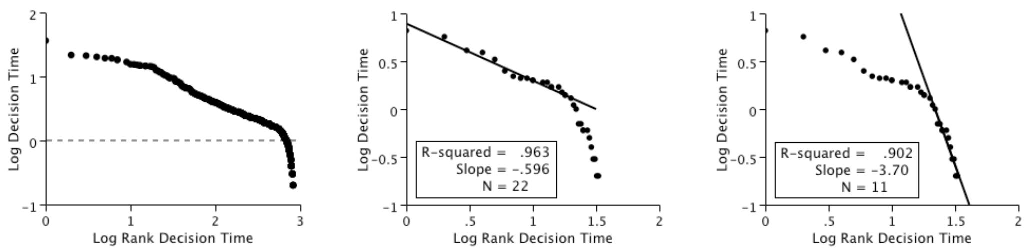
As we will illustrate, the data obtained in this study share a significant similarity with data from a previous study (Hills, et al., 2007) on scale invariance in a HubNet (Wilensky & Stroup, 1999) simulation. The decision times data obtained from a Forager Simulation at the levels of individuals, classrooms, and aggregations of classrooms have the same type of power law distributions and high degrees of correlation as the previous data. Beyond replication of the previous study, we have succeeded in extending our findings to two different HubNet classroom activities, where the same types of scale-invariant behaviors are in evidence at the individual, group, and aggregate levels.

A Discussion of the Data

First we make a case for excluding decision times of less than a second from the data analysis. In the earlier study (Hills, et al., 2007) participants interacting with the Foraging Simulation at intervals under one second were observed to be most intent upon moving quickly through the resource space, apparently paying little attention as to direction, the activities of others, or the distribution of underlying resources. After the simulation, students reported that longer wait times were used to “think about where the resources might be” (p. 231). According to the participants, longer periods of thinking before a move resulted in better success in the problem environment. Our previous analyses demonstrated that participants that performed well in terms of amount of food acquired per step sampled primarily from slower temporal distributions while participants who scored the smallest values of “food per step” sampled from distinctly faster temporal distributions. While this experiment cannot shed light on learning per se, we reason that individuals who were more successful in acquiring resources were more apt to be strategizing and adapting strategies in ways that are related to learning.

In a three-classes aggregation of decision times in the Disease Participatory Simulation the distinctive “elbow” that we have seen previously (Hills, et al., 2007, p. 233) appears at decision times of a little more than one second (Figure 2.2a.). When the data set is (somewhat arbitrarily) separated at the one-second point, the slopes of the least squares fits become -.596 (for decision times greater than one second) and -3.70 (decision times of 0 to 1 second), and the correlation constants for the partitioned data sets improve substantially to .902 and .963 respectively. While the correlation coefficient for the entire data set is .795, partitioning the data into decision times greater or less than one second results in two regions with significantly different slopes and higher correlations.

Figure 2.2a clearly illustrates the elbow in the data, with the dashed line indicating a decision time of one second. Two distinct slopes are clearly in evidence. We saw this same general trend throughout the experiment—in the three different classroom activities shorter decision times lead to steeper slopes and longer decision times resulted in shallower slopes in the least squares fit lines. Zipf plots of the decision times data across the three simulations are quite similar in many respects, and intriguingly different in others. Examining the nuances and subtleties of the data generated in these design experiments must be left for subsequent and more highly informed iterations.



(a) (b) (c)

Figure 2.2. (a) Zipf curve for decision times in a Disease Simulation run, aggregation of decision times from three classes (N = 688). Decision times of less than one second are below the dashed line. (b) and (c), Zipf curve of decision times for a single participant. As in the aggregated data from three classes, two slopes prevail, with the inflection point falling at decision times of approximately one second.

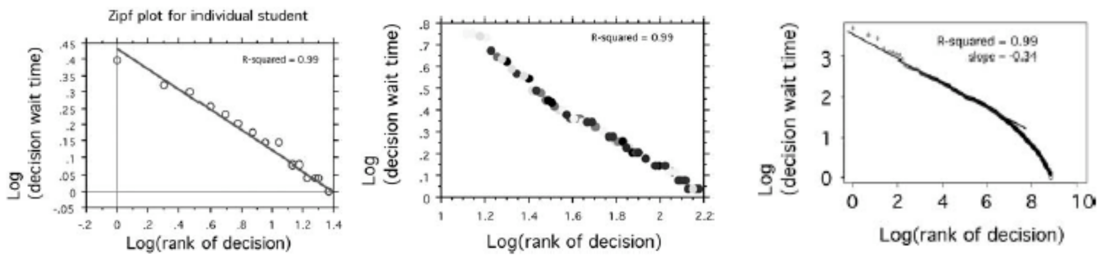
A plausible explanation for the appearance of an elbow in our data comes from studies in human reaction times (c.f., Sanders, 1998), where times to produce an action (e.g. depressing a brake pedal) are investigated and measured under varieties of conditions. Not surprisingly, it turns out that reaction times are dependent upon, among other things, the relative degree of cognitive processing necessary to perform the action. "Simple reaction times" are shorter and do not require cognitive processing, while "complex reaction times" are longer and involve varying degrees of cognitive processing before a reaction to a stimulus occurs (Kosinski, 2006). This is a key distinction being made in the current research, that longer times before making a decision and pushing a HubNet button are correlated with some sort of cognitive processing relative to participation in the simulation.

Reaction times are affected by many factors such as fatigue, stress, and level of engagement, and as such are task-dependent and difficult to measure accurately and reproducibility. Still more tenuous would be trying to characterize the sort of "thinking" or cognitive processing that goes on in such short periods of time. However, the lower limit of complex reaction times seems to be an easier thing to estimate because any complex reasoning must take more time than, say average measured values of simple reaction times, by definition. Lower limits on complex reaction times appear to be on the order of .7 seconds to 1.25 seconds (Green, 2000, p. 213) for automobile braking, and we assume that limits on complex reaction times in a HubNet Simulation would be similar.

For the foregoing reasons, we consider decision times shorter than one second to be simple reaction times, where participants' actions would be limited to simply pressing a HubNet button without spending time thinking about the move. We consider decision times longer than a second to be the time it takes to push the button together with some additional time spent on deciding which of the buttons to push. Further, we expect longer decision times are more likely to provide meaningful information about the subjects' thought processes. We have truncated our decision time data sets at one second, assuming that only longer decision times can provide time for goal-directed cognitive processing.

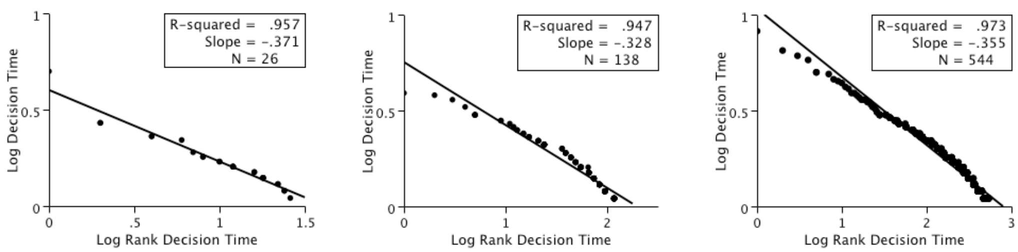
Scale Invariance Across Participatory Simulations Activities

The results of this segment of our research project indicate that, as in the previous research (Hills, et al., 2007), there is a scale-invariant power law relationship (c.f. Zipf, 1949) between simulation participants' decision times and the rank ordering of those decision times. Figures 2.3-2.6 show that in each of three participatory simulations (Wilensky & Stroup, 1999) relatively long decision times happen infrequently and short decision times occur most often. The data indicate that log-log relationships in decision times occur at the individual, classroom, and aggregations of classrooms levels, are linearly correlated, and typically have high correlation coefficients (>.90).



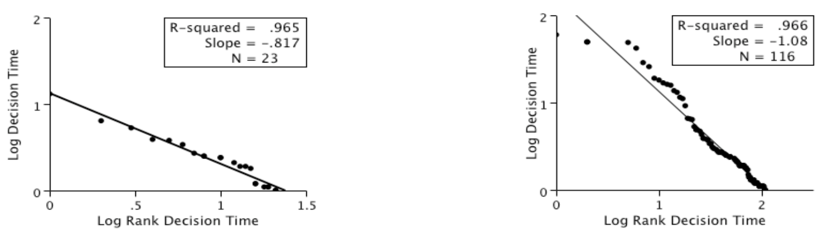
(a) (b) (c)

Figure 2.3 Data from Hills et al. (2007), decision times in Forager Simulations for (a) an individual, (b) a classroom, and (c) an aggregation of simulations from three classrooms.



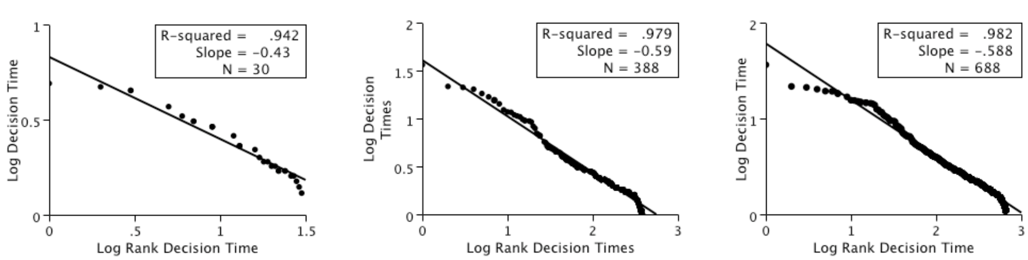
(a) (b) (c)

Figure 2.4 Zipf curves for decision times, Forager Simulation, for (a) an individual participant, (b) a single run in one classroom, (c) aggregated data of three classrooms.



(a) (b)

Figure 2.5 Zipf curves for decision times, Algebra Activity, for (a) one participant and (b) an entire class.



(a) (b) (c)

Figure 2.6 Zipf curves for decision times, Disease Simulation, for (a) an individual participant, (b) a single run in one classroom, (c) aggregated data of three classrooms.

The results reported here replicate and extend the previous findings to two additional participatory activities. Casual observation of the up-front participation space would lead one to believe that the players' activities were quite chaotic—no visible pattern of activity was discernable. However, when viewed in particular ways, the simulation activity was highly organized—producing highly correlated power law relationships. Viewing classroom activity as a complex system encouraged the discovery of a high degree of organization in an apparently highly disorganized activity.

At one level, that is the major finding of this iteration of design research. While it is very tempting to try and interpret the results of this research in such a way as to shed light on learning, neither this experiment nor its predecessor was designed to systematically investigate learning. In the previous work, we saw a positive correlation between success in the Forager Simulation and the time participants took between decisions to move. While that may say something about participants' abilities to successfully adapt, it is still quite far from saying much about common definitions of learning. It is expected that in future experiments we will be able to design activities that more closely approximate activity that can be construed as learning and we may start to develop useful understandings of the subtler differences in correlations.

Discussion

Power law relations have been found in growth patterns of the World Wide Web (Barabasi & Albert, 1999), in the evolution of genera (Reed & Hughs, 2002a), in information theory (Shannon, 1948), in language modeling (Rosenfeld, 1996), in the frequency of word occurrences (Zipf, 1932), in distributions of city sizes (Zipf, 1949), in language development (Cancho & Sole´, 2003) as well as in many other disciplines. In each of the research reports listed above, mechanisms for the emergence scale invariance are postulated, and brief discussions of these mechanisms are provided next.

Scale Invariant Systems

With respect to the World Wide Web, a model of network "growth with preferential attachment" (Barabasi & Albert, 1999) is offered that combines the properties of random addition of nodes and preferential attachment between nodes. This is a "rich-get-richer" (p. 511) scenario, where the web expands through the addition of new nodes that are more likely to attach to nodes that are connected to many others. The authors show that random growth or preferential attachment alone do not lead to scale invariance, it is only when the two properties are combined and allowed to interact that power laws emerge from the system.

Relative to the evolution of genera Reed and Hughes (2002a) propose a model of how species grow and evolve. Theirs is a "... model of macroevolution in which speciations and extinctions are assumed to occur independently and at random, and in which new genera are formed by the random splitting of existing genera" (p. 125). They go on to describe this mechanism (mathematically) as a "homogenous birth and death process" (p. 126) that takes into account that genera have been in existence for varying lengths of time. Reed and Hughes (2002b) extend modeling of this type to several other natural processes.

Shannon (1948) and Rosenfeld (1996) use entropy-based models to account for information transmission in which the ratio of signal to noise is used to get measures of the entropy of information and communication. More recently, Cancho and Sole´ (2003) used an entropy-based approach to model the onset and development of human languages. Their model is based on the simultaneous minimization of speaker costs and hearer costs as language evolves. In their single parameter entropy model, power law relations are generated as optimizations between hearer costs and speaker costs are reached.

Each of these types of explanations may have value for explorations of the scale-invariant behaviors found in classrooms. It may well be that "knowledge networks" can be discovered that are amenable to the types of network modeling that considers growth with preferential attachment or Reed and Hughes' model of the growth of genera. It also seems likely that the sorts of entropy modeling of scale-invariant systems could play a major role in understanding how the behaviors of agents in a system can produce scale invariance. However, it is George Zipf's (1949) "principle of least effort" that we believe will provide a strong basis for useful thinking about classroom learning.

Least Effort and Optimization

The principle of least effort (Zipf, 1949) states that the goal-directed behaviors of humans are governed by the estimation and anticipation of ways of acting that minimize effort over long run. He uses an example of travel between two cities with a mountain range between them. In that case, the traveler may choose any of several paths, the shortest, the fastest, or the easiest. Zipf's principle makes a bold claim—namely that "the entire behavior of an individual is at all times motivated by the urge to minimize effort" (p. 3). While we may not believe that *all* human behavior is governed by attempts to minimize work, we do believe that the notion of optimization may be a powerful tool for research on learning. It seems quite plausible to us that the activity of learners in classrooms is the result of their making conscious and subconscious optimization decisions.

In conjunction with Zipf's principle of least effort, consider John Holland's (1995) modeling of complex adaptive systems (CAS). We have noted above that CAS adapt in order to be more successful in their environments, and in each of the models, Zipf's or Holland's, we see optimization—either in minimizing effort or maximizing "fit." So of all the proposed mechanisms for producing scale invariance, we find this notion of optimization as having the most potential for modeling learning. It seems plausible that in a particular situation, it may be possible to model learners' behaviors as entropy-like calculations between a relatively small number of options.

Optimizations in learning are not apt to be precise as entropy calculations that can be stated in either-or terms, minimizations of speaker costs or hearer costs, for example. We conjecture that learners in classrooms may be simultaneously optimizing across a variety of purposes and goals, perhaps even making "nested optimizations," where a given optimization estimate would be made in the context of a prior optimization. In keeping with our view of learning as a complex adaptive system (Hurford, 2004; Hurford, in review; Hurford, in preparation) optimizations might be seen as being "mutually constitutive"—each influencing and being influenced by others. If our conjectures prove fruitful it may become possible to identify high likelihood optimization "poles" in well-defined classroom situations and to move toward much more mathematized modeling of learner behaviors. Experiments along these lines are expected in the near future.

Reflections

On Theory and Methods

There are certainly a great variety of perspectives on classroom learning, what learning means, and how to promote it. It is always the case that different perspectives on education and learning highlight some aspects thereof and push other aspects to the background. It is only natural to try and simplify our relations with our environment so that we have more cognitive capacity for understanding some subset of associations within it. Each of the theoretical and methodological perspectives has helped educators and researchers to tease meaning out of what Dr. Ann Brown has called the "blooming buzzing confusion" of real-life classrooms (Brown, 1992, p. 141, after James, 1981, p. 462). The purpose of our work is to develop and explore yet another in a long line of perspectives on learning.

Theoretical models underpin research, and the models and questions being asked should point to particular methodologies for the investigation. The theoretical perspectives that frame this research are a combination of cognitive psychology and the complexity sciences. The question that framed the study was whether we would find the same types of scale invariance in decision times of participants across different types of participatory simulations, and design based research methods structured our investigation. The twin aims of design research, building theory in tandem with designing and assessing classroom practice fits our research and dispositions very well.

While this research focuses quite narrowly along "science-based" and cognitively-based lines, that position does not imply that other perspectives (e.g., socio-cultural, radical constructivist, critical) are not also valid and useful tools and means for understanding learning. In fact, systems research explicitly studies systems within systems and so understands well the concept of the embeddedness and mutual constituency of systems. We see that our cognitive-science-based view of learning is happening in multiple contexts and we fully understand that those contexts affect the systems that we are studying.

We are invested in modeling learning as a complex system and in helping to begin the process of infusing complexity-based curriculum at all levels of the educational system. The research reported here has identified a hallmark of complex systems, scale invariant (self-similar) behaviors in the activity of simulation participants. The phenomenon is robust—occurring at multiple levels from a single run of a single individual to aggregations of data from multiple classrooms over multiple runs of the simulation. It is also robust across simulations—we found the same power law relations in three different participatory simulations activities.

The participatory simulations were a vital component of an instructional unit intended to introduce classes of pre-service mathematics and science teachers about complex systems and about learning as a complex system. The hope is that learning about and learning as will be mutually informative and provide the pre-service teachers with both increased understandings and tools for teaching. Relative to pedagogical content knowledge, teachers need to have content knowledge in order to teach a subject well and a secondary goal of our work is to provide teachers and with better understandings of complex systems in general. Our research projects are firmly grounded in building models of learning as complex systems and in teaching learners about complex systems.

On Zipf Curves and Learning

A question that needs to be addressed concerns the relation of the data reported here to learning. Specifically, what does a doubly logarithmic relation between the elapsed time between button presses and the rank ordering of those times have to do with learning? The answer may be, very little actually. But that answer misses the point. The point is that the activity that goes on in classrooms can be viewed, at a theoretical level, as a complex adaptive system, and that means that there exist powerful analytic tools and computer modeling capabilities that can be used to make sense of classroom learning. We have a characteristic fingerprint of complex systems in students' participatory simulations and so we believe we are justified in making the fundamental assumption that the participants' activity constitutes a CAS. We have only begun to experiment with the data collection, the activity design, and the analysis of the data that we have developed the ability to obtain.

On Tools and Technology

The tools that we are using and developing in this series of research projects deserve some discussion. First, the fact that students can be introduced to complex systems and in the same class period can experience being a complex system, in the participatory simulations, is an exceedingly important capability. There are obvious differences between reading about a phenomenon and personally experiencing it, and we see this as an important feature of the classroom experiences we are designing.

The same computer software that hosts the participatory simulations has the capability to manageably collect a wealth of information about the participants' activities and thus provides for multiple and fine-grained analyses of the data. The HubNet/NetLogo (Wilensky & Stroup, 1999, 2002) software captures the state of every simulation variable, the activities of the participants, and the states of the simulation-participants system at very short time intervals, enabling visual replay of the activity that can support reflective post-simulation analyses by researchers, teachers, and students. At the same time, we can obtain the time between decisions/actions by every participant in every simulation in order to discover and report the findings in this article.

Beyond computer-based tools we are developing conceptual models of learning based in complexity, and believe complexity-based modeling of learning is also a technological step forward for the learning theories discipline. Learning systems from individuals to organizations are being described in the literature, and complexity lenses are being used to describe, predict, and explain systems' observable behaviors. Modeling learning as complex system provides a structure and points at methodological approaches for studying learning that is truly unique.

Our particular theorizing about learning as a complex system is structured by John Holland's ECHO model (Holland, 1995), modified somewhat by distinctly cognitive psychological assumptions regarding the models that Holland postulates complex adaptive systems create and use. That structure tells us where to look for how the CAS proceeds (the mechanisms; Holland, 1995), what the process should look like (the processes; Holland, 1995), and that we may expect to find self-similarity and scale invariance in the dynamic activity of complex adaptive systems. The modeling itself becomes a technology in itself that can be used to make sense of learning in new ways.

On Future Directions for this Research

This line of research is in its very early stages, and much remains to be done. Building a theoretical perspective on learning will require generating convincing real-world examples to explicate the LCAS model and how the perspective can be used to further understandings of learning. Viewing learning through complex adaptive systems lenses is helping us to design instruction (see Section Three) and to analyze learning in terms of the models learners use and the memes in their environments that they incorporate into new and emergent models. In keeping with our research methodology, the next iteration of this series of design experiments will attend to theory-building, teaching practice, and to the design of the research methods.

Our proposed theory of LCAS will direct our next investigations. We will try to develop new participatory simulations that can provide better feedback relative to the models participants are using and the strategies that they use in the simulation. Using these new simulations, we will again put the notion of scale-invariance in decision times "in harm's way" (Cobb, et al., 2003, p. 10) by performing analyses similar to those described here on the new data. We will begin to operationalize and look specifically for the processes (flow, aggregation, non-linearity, and diversity [Holland, 1995] that we believe will add to our understandings of classroom learning systems.

From a research methods perspective, many improvements are expected. With continued collaboration with a widening group of members from "the three tiers" (Lesh & Kelly, 2000), we expect develop more sophisticated means for assessing the models learners are using and to improve those assessments such that we obtain clearer pictures of participants' adaptations. We expect that participant interviews will help to inform our understandings of the strategies and models that the learners use. From the perspective of classroom practice we will redesign our introduction to complex systems based on feedback from the current experiment and try and improve on the levels of understanding of systems and particularly learning as a complex adaptive system for future classes of pre-service mathematics and science students.

Closing

Complex systems ideas are becoming increasingly important in the learning sciences (c.f., IJLS, January, 2006) and in developing better understandings of the world in many disciplines. There can be little doubt that human learning may be profitably construed as a complex system. With complex systems frameworks, we can move closer to increasingly mathematized models of learning that have the potential to inform practice. With research in learning about complex systems and in teaching pre-service teachers about complex systems, we have the potential to move school curricula toward important new means for making sense of the world.

SECTION 3

COMPLEX SYSTEMS AS CONTENT AND STRUCTURE IN A LEARNING THEORIES COURSE

The shortage of qualified mathematics and science teachers is a problem of national proportions (National Academy of Sciences, 2007, pp. 113-114) that will require extraordinary and innovative solutions. One very successful program designed to respond to the challenges described in the National Academy's report is the UTeach teacher preparation program at the University of Texas at Austin. The UTeach program, soon to be initiated at several large research universities around the country[1], specializes in recruiting highly qualified science and mathematics students into the teaching profession. Faculty members in the program have dual missions: to prepare excellent next-generation mathematics and science teachers and to generate and disseminate effective research on how to understand and promote learning (Petrosino & Dickinson, 2003). The research reported here was carried out in that spirit in one of the three core classes of the UTeach program—the Knowing and Learning in Mathematics and Science course.

A Discussion of the Course Structure

In Knowing and Learning, pre-service science and mathematics teachers are introduced to theories of learning and to ways in which theories of learning may be used to inform instruction. A key assumption of the course is that science and mathematics can be seen as content that is to be taught *and* as a way characterizing and assessing the learning we intend to produce. An example of content becoming a framework for knowing and learning is that in a biology course we may teach students about the relationship between genetics and a person's height. That biological model becomes a way of thinking about knowing and learning when we posit the same type of relationship between genetics and intelligence quotients. The same natural selection, variation, and mutations that affect height or bone structure are thought of as resulting in a measurable quantity like height, called intelligence. Piaget used the mathematical framework of the algebraic group to theorize about learning (Piaget, 1971; Beth & Piaget, 1966) and we often use statistical mathematics in the form of standardized tests to make important decisions about teaching. A primary commitment of the Knowing and Learning course is to take seriously and make explicit this co-constitutive relationship between content and theories of learning and to purposefully consider the interactions between content and learning in ways that can meaningfully inform our students' pedagogical understandings and practices.

The Knowing and Learning course is taught as a sequence of units on major learning theoretical approaches, both historical and current we generally follow a similar instructional trajectory in each of the units:

a) begin a unit with introductory readings that introduce students to the perspective being investigated;

b) students respond to a set of instructional questions on the readings via the internet and a course web page;

c) an instructor provides lecture and guides discussion of the readings and the perspective;

d) the students engage in some activity chosen to illustrate principles being discussed;

e) learning progress is assessed and the results are used in future course design cycles.

It is often the case that the instructor that is directing classroom discussions is an invited expert in the science or mathematical domain that is being used to structure the perspective on learning under consideration. It is also the case that while we are designing and implementing instruction in the classroom we are simultaneously collecting and analyzing information relative to our teaching, the learning outcomes achieved, and the design of subsequent classroom instruction.

The teaching experiment that we are reporting on was designed in a way that was consistent with the unit trajectory discussed above and was focused on complex systems as a content topic. Complex systems approaches have provided powerful insights and new knowledge in a wide range of scientific disciplines from physics (Prigogine & Stengers, 1997), to biology (Camazine, et al., 2001), to social phenomena such as the growth of the internet (Barbasi & Albert, 1999) and human language development (Cancho & Sole´, 2003). One of the theoretical foundations of our teaching experiment was that classroom learning can plausibly considered to be a complex system (Hills, Hurford, Stroup, & Lesh, 2007) and can be treated as such when designing classroom instruction. The particular complex systems perspective that we have chosen to structure this instruction and experimental investigation originates with the work of John Holland (1995). His is a complex adaptive systems model intended to be applicable to a wide range of complex systems and we have extended Holland's model to take into consideration specifically human behaviors.

In keeping with the overall structure of the course, instruction provided to the UTeach Knowing and Learning students for this teaching experiment included a text-based introduction to complexity and learning as a complex system, asked web-based guided feedback questions on the readings, offered lecture and group discussion opportunities, and facilitated the students' participation in complex systems simulations

(Wilensky & Stroup, 1999). The unit concluded with a third set of web-based survey questions intended to assess and provide insights into student learning.

In previous work (Hills, Hurford, Stroup, & Lesh, 2007; Hurford, in preparation; Hurford, 1998) we have argued that classroom learning at individual and group levels is consistent with several of the formal criteria necessary a system's being classified as complex. A question that we seek to respond to here is "what would considering learning to be a complex system mean for science and mathematics teaching and for pre-service teacher preparation?" Fostering a conversation about complex systems and learning *as* a complex system in the UTeach Program and in the larger teaching and learning community is the primary motivation for our design research program.

Theoretical Underpinnings of this Research

This line of educational research, while on the one hand working to generate theory and models, is at the same time strongly influenced by established theories of learning and by the complexity sciences. In this section we discuss the theoretical foundations of our research agenda.

Cognitive Perspectives

This project shares fundamental assumptions with many established views on learning. The work reported in the present paper is essentially a teaching experiment (Lesh & Kelly, 2000), the first phase of which is structured by a behaviorist/instructionist learning perspective (NRC, 2001, pp. 61-62; Sawyer, 2006, p. 10; Skinner, 1954). In the first phase of our design research, knowledge of complex systems (as curricular content) is viewed strictly as facts and terminology. Learning is taken to mean the ability to respond to questions about the material covered in the supplied text in an acceptable manner.

This admittedly restricted view of learning is augmented in the next phase of the experiment by the cognitivist views of selective attention (Anderson, 1982; Reynolds, 1992) and rehearsal (Craik & Watkins, 1973). The idea here is that if students attend to the definitions and terms of the subject matter and have a chance to actively rehearse the material, they will be more likely to transfer the information from “working memory” to “long-term memory” (Royer, 1986, p. 89). These first two stages of the experiment were designed to effect and enable students’ learning about complex systems by causing the learners to attend to and rehearse text-based concepts.

Borrowing from schema theoretical points of view (Piaget & Inhelder, 1969, p. 4) another assumption of this experiment is that the knowledge instilled in learners is contained in “conceptual ecologies” (Toulmin, 1972) that can structure and give meaning to subsequent complex systems experiences. The assumption here is that once students have created and activated conceptual frameworks of complex systems they will add to and substantially revise those frameworks in the context of experience.

Our theoretical perspective is also firmly rooted in conceptual change literature (e.g., Strike & Posner, 1985, 1992; Vosniadou & Brewer, 1987). We designed the third stage of the teaching experiment based on assumptions related to adding onto and changing learners’ conceptual structures and to the constructivist notion that active participation in classroom-based problem solving extends and improves learners’ - understandings. Thus, describing learning as a CAS is viewed in our research program as a way of situating and combining existing learning theoretical frameworks into a cohesive package for designing effective classroom learning experiences.

Complex Adaptive Systems Perspective

Cognitively based theories of learning under-gird the design of successive stages of this teaching experiment. Those learning-theoretical approaches determine the nature of the project's learning goals, the way we teach, and the formative assessments of the outcomes. However, the design of the *progression* from behaviorist to constructivist teaching and learning was itself directed a priori from a complex systems theoretical perspective.

The model of learning that we hope to instantiate and fruitfully apply to classroom teaching and learning is firmly rooted in the science of complexity. John Holland's model of complex adaptive systems (1995) is the theoretical foundation upon which our entire program of design research is based. With one extension, from tagging rules-sets to persistent internal models, we envision human learning as a Holland-inspired CAS, viewing learners and groups of learners as agents and meta-agents, and learning as adaptation.

We see strong connections between established learning theory approaches and the complex adaptive systems model that we are using. "Schemas" (Andre, 1986, pp. 187-188), "scripts" (Abelson & Shank, 1979) and persistent internal models are closely related—they are both thought of as being activated from some sort of memory, both become active relative to an environmental context, and both help CAS to make sense of their current situations. Knowledge-centered learning theories (Sawyer, 2006, p. 10) see knowledge as the accumulation of appropriately sized "bits" of information, and the CAS model might refer to those bits as building blocks that are acquired through selective attention. We consider persistent internal model (PIMs) to be stored in "long-term memory" (Royer, 1986, p. 89) and our CAS learning approach sees real time internal models (RIMs) as being located in "working memory" (p. 89). The complex adaptive systems perspective provides a unique venue for combining multiple and in some ways discordant learning perspectives into a cohesive approach for thinking about learning and for designing instruction.

Learning as a Complex Adaptive System

When we think about how learning happens, we assume the entity that is doing the learning is a complex adaptive system. We further assume that the learning system can be modeled in agreement with John Holland's (1995; see also Hurford, 2004; Hurford, in review) perspective, and that we should be able to identify what Holland calls attributes, mechanisms, and properties of the complex system we have chosen. When using systems approaches it is important to specify the system being studied. While there are many possible candidates in schools that could be considered to be learning systems, for the purposes of this article we choose to consider learning at the level of individual students, where the interacting agents are concepts and conceptual structures. We argue that viewing learners, as CAS, through Holland's lens will provide us with powerful new "sightlines" (Hamilton, 2007) into, and knowledge about, learning.

We propose a more sophisticated and specifically human mechanism that extends Holland's (1995) tagging rules (p. 14) and refer to this revised mechanism as "persistent internal models" (PIMs). Persistent internal models are more than just tagging rules sets—we envision them as whole conceptual structures (mental models) that possess both declarative and procedural information. PIMs have the same *function* as tagging mechanisms—they direct attention and enable attention to and acquisition of building blocks (Holland, 1995, p. 34) for composition into internal models. Once a persistent internal model is activated it enables the complex adaptive system to form a real-time internal model (RIM) of the situation that in turn enables the complex adaptive system to make sense of and adapt to its environment.

The RIM is the same as Holland's "internal model" (1995, p. 31), and we use the "real-time" distinction to discriminate between preexisting persistent internal models and these models that emerge in the moment, in response to a particular situation and context. Alternative PIMs may be retrieved and activated as situations change, and current RIMs will morph into new ones. Depending upon the relative success of the complex adaptive system's activity, previous PIMs may be restructured and the new persistent internal model stored for future reference. We consider this restructuring to be a component of learning. We believe that our adapted CAS model of learning provides a powerful tool for designing and implementing classroom instruction, because it tells us where to look for learning (in RIMs and PIMs), tells us how to proceed (by appropriate distributions of building blocks), and points to sampling internal models in order to conduct formative and summative assessments.

To summarize, complex systems are aggregations of agents, in individual learners we see these as concepts and conceptual structures, acting independently that create larger scale patterns of activity when viewed from outside the system. Complex adaptive systems are a subset of complex systems that have the ability to change themselves and their behaviors in response to feedback from their environment. John Holland (1995) generated a model of a more-or-less universal complex adaptive system, which we use to study learning. We have extended Holland's mechanisms in order to take into account the human mechanisms of persistent internal models that develop into real time internal models. Throughout this design research experiment we use Holland's CAS model to inform *both* how we think learning occurs, and how we design classroom-based learning activities.

The Teaching Experiment

The research question driving this iteration of teaching experiment was the following:

> How do the pre-service teachers perceive the mechanisms of the adapted Holland model of complex adaptive systems relative to their participation in the Disease simulation?

The subjects were 34 pre-service secondary mathematics and science teachers enrolled in a domain-situated learning theories which is required of all students participating in the UTeach Program and the University of Texas at Austin. The study included about an equal mix of men and women and all of the students were pursuing bachelor's degrees in the fields of mathematics and science as well as studying to become teachers. The sample was strictly a convenience sample, questions of generalizability to other populations of students and teachers were not addressed in this study. The purposes of this early iteration in our design research agenda were strictly exploratory: we wanted to begin the process of designing complex systems-based courseware, test the introductory approach, engage the students in complex systems simulations, and obtain a measure of the students' understandings of the subject materials after teaching the unit lesson. The CAS content was engaged in ways that are consistent with the overall design of the Knowing and Learning course—focusing on the interactions between domain content and theories of knowing and learning in ways that can inform teaching. The experiment unfolded in a series of three phases.

Phase One

Before the students could reasonably respond to our research question it was necessary to introduce them to complex systems terminology and ideas. They were provided with a journal-type article (Appendix A) to read before attending a lecture on the topic in the following class period. The article, authored by one of our researchers, was an introduction to complex systems in general and then to our version of Holland's complex adaptive systems model (1995) applied to learning. The students read the paper and responded to a set of questions designed so that the learners would rehearse and reinforce the ideas covered in the reading. The questions were presented on an Internet-based web page and students' written responses were captured electronically. This introduction to complexity was intended to be very knowledge-centered and instructionist (Sawyer, 2006, p. 10) in approach. We wanted to familiarize students with the major ideas and concepts and to try to get them to commit the more salient ideas to memory.

The class meeting in this phase featured a knowledge-centered lecture on the assigned reading. At the conclusion of the first phase lecture the students engaged in the Forager Participatory Simulation (Hills & Stroup, 2004; Hills, Hurford, Stroup, & Lesh, 2007) whole-class activity. In this activity, participants "forage" for clumps of hidden food, simulating the behaviors characteristic of animals searching for food. The purpose of having students participate in this simulation was to familiarize them with the HubNet (Wilensky & Stroup, 1999) interface and to give them an introduction to being an agent in a complex adaptive system.

Phase Two

The next reading assignment was a chapter from Peter Senge's book, the *Fifth Discipline* (1994, pp. 27-54). The chapter is a discussion of a simulated commodities distribution supply chain, and presents a classic complex systems activity that is widely used in business school classrooms. The purpose of the reading was to deepen students' knowledge of complex systems and to illustrate how complex systems can produce effects that the system, rather than the participants, is responsible for. This reading, as with the previous reading, was accompanied by a set of internet-based response questions. While the Phase 1 reading on complex adaptive systems was new to the course, the Senge reading and questions were the same as those that had been used in previous semesters. This provided us with an opportunity to see how students' understandings of systems may have been affected by the first complex systems reading assignment.

Phase Three

The third phase of this design experiment included a discussion of the Senge reading, a brief review of complex adaptive systems ideas and terminology, a whole-class participatory simulation focused on disease transmission, and a related set of web-based response questions. The bulk of the time in Phase 3 was spent doing multiple runs of the Disease Simulation (Wilensky & Stroup, 2002). Following the simulation activity the students were asked to respond to a third set of web response questions intended to obtain information relative to the research question—what aspects of the mechanisms of the adapted CAS model did the participants recognize in their simulation based activity? The questions asked of the students, their responses and our commentaries for each of the three phases of the experiment are reported below.

Methods

After immersion in the answers to the questions we had asked we developed scoring rubrics for assessing students' responses. The first rubric (Table 3.1) was based in knowledge-centered learning theory and was a way of scoring the students' responses—either the students supplied the answers we thought best, or they did not. The questions asked relative to the Senge (1994) reading were a mixture of knowledge-centered and more open-ended questions. While the Phase One reading on complex adaptive systems was new to the course, the Senge reading and questions were the same as previous semesters. This provided us with an opportunity to compare our responders with prior students to get an idea of how the present class's understandings may have been affected by the first complex systems reading assignment.

With the exception of the first question asked, the Phase Three web-response questions were much more open-ended. The first question was knowledge-centered and was intended to assess what and how much about the mechanisms of complex adaptive systems (the Phase One reading) the students could recall. The remaining questions were designed to shed light on how the simulation participants were making sense of the mechanisms of CAS and whether or not they were engaging with the ideas in general. The web questions for this phase of the experiment were more intended to provide the researchers with insights into students' thinking about complex systems than they were to be summative assessments of student knowledge.

Table 3.1

Web-Based Questions and Scoring Rubric Relative to the Complex Systems Reading.

1. What is a system as defined in "Thinking of Learning as a Complex Dynamical System" (p. 3)?

    Rubric Scoring: Award 1 point each for mentioning "some number of agents," "patterns of behavior," "hierarchical levels," "tagging-type rules." (4 points)

2. Give 2 examples of complex systems from biology.

    Rubric scoring: Multiple answers to this open-ended question were permissible. (2 points)

3. Could traffic flow out on Interstate 35 be considered to be a system?

    Rubric scoring: "Yes" was the only accepted response. (1 point)

4. List the 10 components of John Holland's (1995) model of a complex adaptive system (p.11)

    Rubric Scoring: Typing in of all ten components was the only acceptable response. (1 point)

5. What is a persistent internal model (PIM, p. 30).

    Rubric Scoring: Response should include—A persistent internal model is a conceptual structure, stored in long-term memory, and activated in response to the CAS's current context. They are rules-like schemas that provide a basis for the

CAS's parsing its local environment by selecting "building blocks." The persistent internal model, under the influence of the new building blocks, becomes the real-time internal model. (0 → 4 points)

6. What is a real-time internal model (RIM, pp. 30-31).

Purpose: This is a rehearsal-directed question intended to get students to describe RIMs in their own words.

Rubric Scoring: Response should include—A real-time internal model is an adapted persistent internal model. The PIM directs attention to building blocks, filling gaps or replacing elements, resulting in a new real-time internal model. The RIM then enables adaptation in the complex adaptive system such that it is better able to act in its environment. Based upon subsequent feedback, the RIM may be further adapted. Real-time internal models may be stored in long-term memory as persistent internal models. (0 → 4 points)

Results

The results for the first iteration of this teaching experiment were very encouraging. In general, students' responses to the first set of web questions reflected reasonably good engagement with and understanding of the notions of complex systems. Their responses to the knowledge-centered questions demonstrated that the students had become familiar with the fundamental notions of complexity science and learning as a complex adaptive system. They participated in the classroom activities and made good efforts at making sense of the ideas and translating those ideas into conceptualizations of the real world. Beyond that, we, as researchers, made significant progress in developing a unit plan and activities for helping teachers to understand the complexity sciences, a first step on the path of introducing complexity-based ideas and tools to the classrooms of tomorrow.

Phase One Results

The students read the opening paper (Appendix) and responded to the knowledge-centered questions asked of them. As Table 3.2 indicates, the average response by students to a question asking for the definition of a “system” contained about three of the four points we considered necessary to adequately describe a system. Understanding the terminology and interactivity of systems is assumed to be prerequisite for developing a working knowledge of complexity and these students demonstrated strong beginnings in that direction. Questions two and three were intended to see if students could identify biological systems in natural and human-agent systems, and nearly all of the responders were able to do so. The fourth question was intended to simply have students “rehearse” (Pintrich, 1999) the components of a complex adaptive system and all students did. Questions five and six were more open-ended than the previous three but still contained direct links (page references) back to the assigned reading, thus maintaining a knowledge-centered flavor. Question five asked the students to discuss attributes of “persistent internal models” and had four assessment criteria. Respondents did a fairly good job of answering this question, with about two-thirds of the students naming at least half of the components we were looking for. The results for Question 6, the question relating to “real-time internal models” were similar, with again about two-thirds of the students mentioning at least two of the four assessment criteria.

Table 3.2

Phase 1 Scoring Rubric Results—Thirty-four Participants.

Question	Results
1	Twenty-two students listed three or four of the rubric criteria.
2	Thirty students were able to correctly name 2 biological examples of complex systems.
3	Thirty-two of the thirty-four students gave the recognized traffic flow as a complex adaptive system.
4	All students responded correctly, that is each student "rehearsed" the list of components.
5	Twenty-three of the students mentioned at least two of the four assessment criteria.
6	Twenty-two of the thirty-four students mentioned at least two of the four assessment criteria.

Phase Two Results

The second phase of this teaching experiment consisted of the Senge (1994, pp. 27-54) reading and a set of web-based questions relative to the reading and to the nature of systems. One of the more interesting results occurred in this phase of the experiment. None of the student responses to the Senge-focused questions made mention of the terms or ideas in the complex adaptive systems reading of Phase 1. Responses to the web questions were undifferentiable from those of the previous semester—apparently the students were making not connections between the two complex systems perspectives.

The complex adaptive systems model described in the Phase 1 reading (Appendix A) on an agent-based model of CAS—the focal point of the model is the adaptive agent. The Senge (1994) reading focuses on an aggregate-based view of complex systems where stocks and flows are driving metaphors. The fact that the students did not find connections between the two readings is actually consistent with a similar disconnect in the research literature associated with complex systems. While some have argued for the complementarity between the two approaches (Chen & Stroup, 1993) most complexity research tends to rely on either on agent-based approaches (e.g., Holland, 1995; Wooldridge, 2002) or aggregate-based approaches (e.g., Sterman, 2000; Forrester, 1968). It seems reasonable that if such a dichotomy appears in the research literature, one might expect to see the same cognitive disconnect in students newly introduced to the ideas. Further investigation of this phenomenon and making explicit the distinctions and potential complementarities of these two types of modeling is a new goal for future iterations of this project.

Phase Three Results

The results from the third set of web-based questions were rich with information about how students were making sense of complex systems and learning. For the current purposes we include evidence and discussion of how the students made sense of learning as a complex adaptive system—the extended model that we presented them with. We also focus on how the persistent internal models the students identified were adapted, and offer some insights and discussion of the students' understandings of the Disease Simulation.

The basics of the model

Most of the students responded to the first question we asked in ways that indicated that they had understood the basics of the three mechanisms as extended to learning as a complex adaptive system, persistent internal models (PIMs), building blocks, and real time internal models (RIMs). While not all of the students' responses were as complete as these, we provide two excerpts to illustrate how they were thinking about the mechanisms.

> *PIMs are the platform or the "basics" that preexist in your mind. The building blocks are things in real time that you can add to your platform to tweak and enhance your PIM but because it is in real time it becomes a RIM. An example is a naked Christmas tree as a PIM, building blocks are the ornaments and tinsel and lights and the new overall dressed tree is the RIM.*

> *a persistent internal model represents our accumulated understanding of a generalized situation or class of situations… in accordance with the pim we have conjured up, we choose what features in our environment will serve as building blocks in developing our real time internal model. this rim is formed by taking the pim as a kind of template and then continuously incorporating novel elements as our environment changes and as we are able to more accurately understand it. it is transient, constantly in flux…*

These student responses really get at the fundamental meanings of the mechanisms and serve as excellent examples of the kinds of understandings of learning as a complex system that we want future mathematics and science teachers to have. They speak to the nature of learners' internal models, their pre-existing "cognitive structures," to the building blocks that are sampled from the learners' environment, and to the real-time models that learners use to make sense of and adapt to that environment.

Adaptation of students' internal models

In this final phase of the experiment a student finally made reference to the Senge reading and a systems-based approach to thinking about the spread of disease. This student responded to the question about their initial (persistent) internal model of the Disease Simulation in this way:

> *After the Beer Game reading, I thought that this might be related to the idea that though a person's tendency would be to be reactive in the disease simulation, the "do nothing" approach* [see Senge, 1994, p. 47] *was actually more effective in avoiding infection for me. That reactivity was a PIM that came to mind.*
>
> Which became the following real time internal model:
>
> *I thought that the persistence of the patterns of the system's results was made evident. Initially, I thought that how I reacted could actually make a difference.*

This student started with an PIM that relates to an advantageous strategy discussed by Senge—doing nothing in the face of crisis in the system. In the end, the student suggests a much deeper understanding when they discuss the patterns of the system and echo Senge's message that system failure can sometimes be natural outcomes of the system and not the fault of individuals.

Another example illustrates adaptation toward an increasingly "mathematized" understanding of the spread of disease. One student described the persistent internal model of the Disease Simulation in terms of "the smooth curves we work with in math classes." After the simulation they had this to say about their emergent real-time model of disease spread. "we saw how the disease actually spread. [We] looked at the graph and adjusted our expectations to what was actually happening." This student's real time model of disease propagation went from an idealized model of a smooth curve to a contextualized model with more texture and uncertainty.

Both of these examples of changes in students' internal models indicate the types of changes in understandings that we seek to foster in this learning experiment. These are the sorts of persistent internal models—focused on mathematical representations and systems perspectives—that make more substantive foundations for the types of learning that we would like to encourage. Having identified the norms and some more desirable alternative starting conceptualizations (PIMs) we believe that we will be able to revise this activity in such a way as to improve the odds that students will emerge with more mathematized and science-focused understandings.

On the one hand, we see students attending to notions of pre-existing and emergent mental models and at the same time we see changes in their internal models toward more systematized and mathematized ways of viewing science content knowledge.

Patterns in students' learning

Looking at the students' responses this third set of complexity-based questions as a whole, two categories of thinking emerged as typical. One category centered on what we came to call "Contagion," where the students were paying attention to things, communicability or infection rates for example, associated with the transmission disease through a population. The second major category focused on the “Game” aspects of the HubNet simulations in one way or another. Responses along these lines centered on the video-game nature of the participatory simulation, where students discussed their internal models of the simulation mainly in terms of the human-computer interface. Certainly the former category was preferred.

Beyond the Contagion category, responses from a few students demonstrated rarer features that we see came to see as significantly better learning goals. The examples given in the section above are the types of student responses that we want to nurture and encourage. The results of this experiment have pointed to new and clear changes for the next—realizing what types of persistent internal models that we can expect the students to activate, we can design instruction to skew the results in our favor. The proposed instructional change here could be something as simple as direct and explicit discussion of the types of conceptual frameworks students have evidenced in previous iteration.

Summary and Conclusions

The UTeach Program at The University of Texas at Austin is an innovative pre-service teacher preparation program that focuses on recruiting science and mathematics students into education and careers as teachers. When these students graduate they have four-year degrees in their primary discipline as well as secondary education certification. This teacher preparation program has proven to be very successful and this article discusses a component of that success—a core requirement course called Knowing and Learning. We performed and report on here a teaching experiment in conjunction with that course that was congruent with the general design format of the course and that focused on complex sciences as content material and as a way of thinking about learning.

In keeping with the overall nature of design experimentation we sought to increase our theoretical understanding of learning and instruction as well as to design and iteratively improve a teaching unit on complexity science. As is common in Knowing and Learning, we invited domain expert on complex systems and learning to help design and execute the unit lessons and to critique the learning outcomes. Student teachers were engaged in learning activities that unfolded in phases and that progressed from behaviorist to cognitively oriented to constructivist and even to complexity-based learning approaches. At each stage the definition of learning, the types of learning activities, and the nature of assessment change in significant ways. It is an underlying assumption of this research that the combination of multiple means and modes of learning and teaching will provide unique opportunities for learning and for designing instruction.

The results of the experiment were encouraging. Students learned fundamental complex systems ideas and terminology and were successfully introduced to multiple complex systems-based approaches, including a very new one that focuses on learning *as* a complex system. Our intent was to introduce the pre-service teachers to complexity as a subject domain, and resonant with the overall format of the Knowing and Learning course curriculum, as a way of structuring the way we looked at learning. The unit concluded with a highly interactive participatory simulation (Wilensky & Stroup, 1999) and a web-based structured interview that demonstrates that students could and did form essentially correct and potentially useful conceptions of complexity and complex adaptive systems learning.

We have provided several examples of the ways in which scientific and mathematical knowledge structure established ways of viewing learning. In this article we offer a new way of making sense of learning and teaching through a complex systems lens. For an *individual* who is considered as a complex adaptive system (Holland, 1995), learning becomes a sequence of events that begins with retrieving a pre-existing model related to the situation at hand, sampling that situation for specific details, and using those details to create a real-time model of the situation that enables the learner to anticipate and act. Considering *classroom* instruction from the CAS perspective enables the seamless coordination of various and sometimes discordant theories of learning in ways that may actually attend to that chronic complaint from pre-services teachers in learning instruction classrooms—"don't tell me about theories, I just want to know how to teach!" From making sense of learning in an individual to offering structure for designing classroom instruction, complex adaptive systems models provide powerful new lenses for education research.

END NOTE

[1] https://uteach.utexas.edu/index.cfm?objectid=371426A3-B525-84BA-21AC3056D03A24A3, retrieved March 2, 2007.

APPENDIX

Learning as a Complex Dynamical System

Introduction

Educational research and theory building are increasingly being influenced by the sciences of complexity, non-linear mathematics, and systems theories. The primary purposes of this paper are to provide the readers with a brief introduction to dynamical systems[1]-based models of learning and to discuss a particular model that represents a step forward in "mathematizing[2]" complex systems models of human learning. This work is an intentional effort to try and use the mathematical to structure the social, to use increasingly mathematized (and "science-ized," if you will) structures to inform our perspectives on classroom learning. The paper opens with the scientific, mathematical, and learning-theoretical contexts that situate complex systems modeling of learning and it goes on to discuss a particular model that seems to hold significant promise for making

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[1] For a quick description of my working definitions of "complex" and "dynamic" systems, please see the Appendix.

[2] Lesh and Kelly (2000), have this to say about the meaning of "mathematization":

> [M]athematics entails seeing at least as much as it entails doing.…one could say that doing mathematics involves (more than anything else) interpreting situations mathematically; that is, it involves mathematizing. When this mathematization takes place, it is done using constructs (e.g., conceptual models, structural metaphors, and other types of descriptive, explanatory systems for making sense of patterns and regularities in real or possible worlds). (p. 224)

The Appendix also has a bit more to say about mathematization.

sense of learning at many levels of analysis from small-scale knowledge reorganizations in a single person, to learning in small groups, to learning in larger groups such as classrooms or corporations. Although this branch of learning theory research is quite new, data that illustrate aspects of complex learning will be presented. The paper also offers a brief discussion of the connections between this new theoretical model of learning and established cognitively and socio-culturally based learning theories (like the ones you've been studying in this class) and points toward ways the approach can be used by real teachers.

First, What's a System?

From the standpoint of complexity theory a system is considered to be composed of some number of agents or elements that act in some fashion and whose behavior is characterized by distinct patterns of behavior when viewed from one or more organizational layers "up." The important thing from a systems point of view is to be very clear about the boundaries of the systems and the identity of the elements that it is composed of. So, for example, individual ants could be thought of as the agents in the complex system that is the anthill, or individual bees could be the agents in the complex systems we call beehives.

Well-defined systems then, are considered to be composed of lots of individual agents that are in turn thought of as acting according to some internal set of rules. Those rules, in coordination with feedback from the environment, determine how an agent will behave, and the organizational structure of the system (anthill, beehive) is the dynamic result of the combined activities of all of the agents in the system in a kind of interactive dance. Very much like a "community of practice," (Lave & Wenger, 1991), the agents, the environment, and the system are mutually constitutive—each affecting and having an effect on the other as they evolve. Not all systems are dynamic or complex, but it is the thesis of this paper that learning of the sort that goes on in classrooms can be reasonably and usefully characterized by complex systems-based models.

Examples of complex systems in nature—self-organization

The opening chapters of *Self-Organization in Biological Systems* (Camazine, et al., 2001) provide an excellent description of many interesting and complex biological systems. From the "synchronous flashing of fireflies" (p. 8), to the ways that fish form into schools, to pattern formation in slime molds and bacteria, Camazine and his fellow authors present many clear examples of emergent complex systems in the natural world. The focus of this book is on one element of the theory of how these systems emerge—self-organization:

> Self-organization is a process in which pattern at the global level of a system emerges solely from numerous interactions among the lower-level components of the system. Moreover, the rules specifying interactions among the system's components are executed using only local information, without reference to the global pattern. (p. 8)

Beyond describing, illustrating, and closely studying many examples of complex systems, Camazine et. al, provide specific insights into another important feature of complexity systems—self-organization—and I will have more to say about that idea next.

Self-organization the process by which pattern emerges from the activity of individual agents in a dynamic system. The agents act solely of their own accord, making decisions based on their own internal rules systems and in response to their immediate environmental situation. At the next level up, which, in the case of bees would be the hive, a pattern emerges (the structure of the hive) that could not be anticipated from a close study of the activities of the individual agents. That's a complex system.

Another example of self-organization in biological systems is the way that geese organize themselves to form a V pattern in their migratory flights. Contrary to popular belief, there is no "head goose" directing the other geese in the flock to behave in a certain way. Each goose responds to its environment, probably trying to reduce wind resistance and perhaps keep a certain distance from its nearest neighbors, and the pattern of flight emerges naturally from the mutually influential activities of each of the agents.

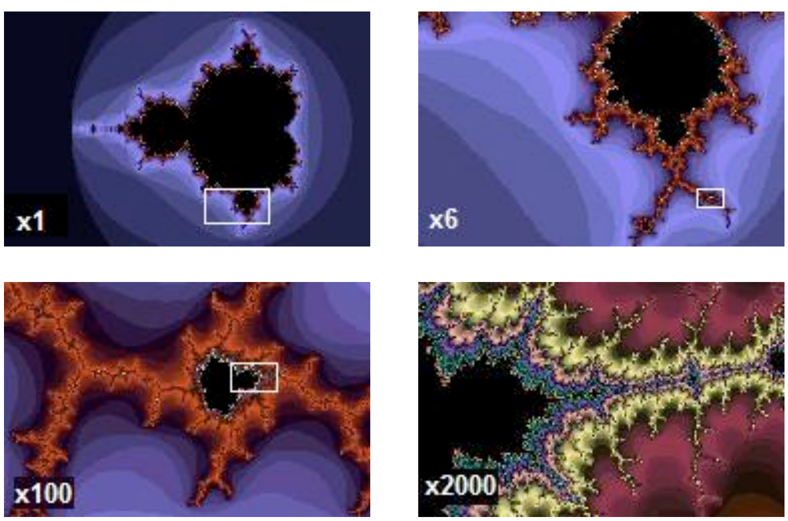
The concept of self-organization has implications for classroom teaching and learning. When students are intensely engaged in, say, a constructivist inquiry activity, you can actually "see" that intensity. Students are totally "on task," and completely engaged in their own learning. Although it will be a difficult thing to prove, it seems reasonable to at least this author that what you're seeing is students' self-organizing their knowledge. In particular, the learners, and not some outside force, such as a teacher, are shaping their understandings. In the same way that those beautiful Vs of geese form without the benefit of a "head goose" ordering the other birds around, I claim that the most engaged and important learning that students do is very likely a self-organized behavior. An interesting implication of self-organization and learning that may be empowering to teachers is that control of the students' learning can and should be a shared responsibility.

A specific type of complex system—SOCS

An important subset of complex systems is the one called "self-organizing critical systems" (e.g., Bak & Chen, 1991). This variation on the complex systems theme is a larger construct than just self-organization and includes several other features that have important implications for learning theory. The system described by Bak and Chen was the formation of a sand pile like the one that forms in the bottom of an hourglass when the glass is turned over. Here again, the pattern that emerges—the sand pile with its characteristic cone shape—is the net effect of the self-organizing activity of thousands of agents (grains of sand). While moment to moment details of which sand ends up where is a series of random events and quite unpredictable, the conically shaped pile (and even its "angle of repose") is completely predictable—that same shape emerges every time the hourglass is turned over. Now rather than being decisions per se, as in the case of a human learner, self-organization in this case just means that every grain behaves according to a particular set of characteristics that act like rules (for example, size, shape, weight, roughness) and determine how the grain moves and how it affects the other agents and the overall system. Again, the interrelationships between sand grains and the system are mutually constitutive.

In addition to the idea of self-organization, Bak and Chen's (1991) variation of systems theory provides us with two other useful notions for making sense of these types of systems. The first notion is that these self-organizing critical systems (SOCS) are said to be "self-similar." So, in the case of the sand pile that grows upward in the bottom of an hourglass, the fallen grains of sand will re-organize every time another grain of sand drops onto the top of the pile. These re-organizations, or little avalanches of sand, occur continually as the sand pile grows. Some of the avalanches may be very small, consisting of only a few grains of sand that move a short distance down the pile and then stop. A much smaller number of re-organizations may be relatively large and consist of hundreds or thousands of sand grains that avalanche and tumble all the way to the bottom of the pile. Self-similar here means that one re-organization (avalanche) "looks" very much like another, with the only real difference being that of relative size. This phenomenon is also referred to as "scale invariance" which means that the system "looks" the same at various scales or levels of magnification. A good illustration of self-similarity can be seen in "fractal patterns" where, again, the pattern of pixels in the fractal looks similar at many levels of magnification.

Figure 1.



The same fractal image at 4 levels of magnification. (Retrieved from http://en.wikipedia.org/wiki/Fractals, 12 April 2006.)

Another illustration of SOCS theory provided by Bak and Chen (1991) is their discussion of "sensitivity to initial conditions." Going back to the avalanches that occur as the sand pile builds, what determines whether an avalanche will be large or small are the "initial conditions" that existed just before the grain of sand hits. In this case, the initial conditions are the relative instability of the sand pile that resulted from the last reorganizations and things like the speed, rotation, and size of the falling grain of sand. If the sand pile is very unstable, with many grains of sand perched precariously on the edges of others, one more grain of sand may cause a very large avalanche. If a new grain of sand hits the pile when it is relatively stable, the size of the resulting avalanche may be very small. It may even be that a single grain of sand will drop and cause no avalanche at all—it could just come to rest perfectly balanced on top of another grain of sand! In that case, it is safe to say that the pile is likely to be less stable after the event, and set up conditions conducive to a larger re-organization later. In any event, the consequences of every falling grain event is heavily dependent on how previous events have affected the relative stability of the sand pile.

To summarize, self-organization is the property of complex systems that describes their ability to produce very clear patterns of activity at one level through the independent activities of agents at a lower level of organization. Self-organizing critical systems are sensitive to initial conditions, which means that future states of the system are highly dependent on earlier states of the system. Small differences in the initial conditions can result in large differences in subsequent reorganizations. SOCS are self-similar, meaning that their activity at one level of observation looks very much like their activity at a different level of "magnification" as it were.

At this point, you are probably asking yourself what all this SOCS business has to do with learning in individuals and classrooms, and it is this question that I will attend to next.

SOCS in classrooms

Self-organizing critical systems theories are strikingly congruent with many aspects of cognitive learning theories and with constructivist philosophies. One can view learning itself as a SOCS, where, for example, the patterns of learning in a classroom emerge from the self-organized behaviors of individual learners.

Each of the individual "agents" in the classroom is making her or his decisions and knowledge re-organizations in a direct response to local conditions and personal initial conditions. Constructivists and cognitive learning theorists studying reading have known for a long time that what children learn is highly correlated with the knowledge and experiences that they bring with them into the classroom. What some call prior or background knowledge complexity scientists would call a subset of the "intitial conditions" of the system.

At another level, knowledge structures (like those discussed in the expert-novice chapter of *How People Learn,* NRC, 1999) within an individual reorganize as a function of reflection and experience. Piaget's notions of assimilation and accommodation can also be viewed as re-organizations of knowledge. The similarities between the image of the small and large avalanches on sand pile in an hourglass and small and large scale re-organizations of learner's knowledge seems quite useful. In another paper I have discussed (Hurford, 1998) the nature of these types of re-organizations (what are sometimes called "ah-hah moments") and how they are a feature of learning. I would be happy to provide you with a copy of this paper upon request.

The last feature of SOCS that I will discuss in relation to human learning is this notion of stability and instability of structures. Just like the falling grains of sand in an hourglass result in changes in the relative stability of the sand pile, one can imagine that new knowledge and experience can cause changes in the stability of learners' knowledge structures. At this juncture, I see a strong correlation between SOCS theory and Piaget's "disequilibrium." Both describe a state of the system that is very unstable and that unstable state is viewed as a necessary precursor to change in the system.

Now all of these comparisons between self-organizing critical systems theories and cognitive and constructivist views of learning are to this point strictly metaphorical. However, even at this relatively low level of mathematization the construct of SOCS offers teachers and learners alike useful ideas about how and why learning may happen. For example, SOCS theory provides a plausible reason for *why* activating prior knowledge is important for learning. It also lets learners know that their understandings will necessarily go through chaotic and stable phases, and that the chaotic phase, where a learner feels like he or she suddenly knows nothing at all about the subject he/she has been studying all week is actually a re-organization phase of knowledge development, and that that is a good thing. It's good because it lets the learner know that progress is being made and that a new level of stable understanding is likely to be just around the bend. It tells us that small knowledge structure re-organizations are happening all the time and that a big ah-hah!, where lots of things fall into place, can happen at any time and that the only way not to learn about something is to not engage with it.

So, if we just think about learning as self-organizing, systems theory helps us to develop constructs, to mathematize our thinking about learning. If we go a step further, and think about Bak and Chen's (1991) SOCS model, we have mathematized our construct of teaching and learning still further. At this level, we have taken into account the natural behavior of complex systems and extended our metaphor so that it covers more ground and considers plausible reasons for how and why learning happens. Furthermore, complex systems constructs have the ability to combine insights from multiple theories of learning into a more cohesive and understandable whole. There is another layer of increased mathematization that is actually the central focus of this paper, and it is this model of learning that we turn to next.

Learning as a Complex Adaptive System

Perhaps the most promising discussion of complex systems for theory-building about classroom learning is John Holland's 1995 book, *Hidden Order*. In this work, Holland takes on the task of attempting to lay out and define a prototypical, complete, and universal complex adaptive system. Holland's model is focused on complex systems that can adapt, based on feedback from their environments, in order to be more successful at. He believes that it is possible identify a set of ten attributes that *all* complex adaptive systems (CAS) can be seen to possess (Holland, 1995, pp. 6-10). He describes CAS as being fundamentally characterized by the presence of agents, meta-agents, and adaptation, and says that these three attributes can be understood and studied in terms of the “seven basics”—four “properties” (aggregation, nonlinearity, flows, and diversity) and three “mechanisms" (tagging, internal models, and building blocks) (Chap. 1).

The attributes—adaptation, agents, that can be seen as aggregating into meta-agents—and the mechanisms and properties serve at least two purposes. First, they can be used in deciding whether or not a system under study can indeed be thought of as a CAS, and second, the features, mechanisms, and properties provide analytical tools for investigating the nature of that system. Holland’s book represents an ambitious effort to create a *general* model of complex adaptive systems, a class of things that can profitably be used to study learning in many contexts. A brief discussion of the attributes, mechanisms, and properties is offered below.

Attributes

First, Holland (1995) sets out the notions of agents, meta-agents, and adaptation as being present in all CAS and necessary if a system is to be categorized as such. The agents, “active elements” that are “diverse in both form and capability” (p. 6) are seen to act, to behave, as if they were responding to an internal set of rules. He is quick to point out that these hypothesized rule sets are not *necessarily* the rules that govern the behavior of the agents, or that rule sets are actually what determine the agents’ behaviors. Instead, thinking of the agents as rules-driven in this way provides “a convenient way to describe agent strategies” (p. 8).

Next, Holland employs a common tactic in systems analyses by moving up a level and discussing "meta-agents" as being a way of thinking about "*what CAS do*" (p. 11, italics in the original)—meta-agents are higher-level agents whose behavior patterns result from the aggregated combination of the behaviors of the agents a level down in the hierarchy. In the same way, meta-agents can aggregate into "meta-meta-agents," thus creating "the hierarchical organization so typical of CAS" (p. 9). To summarize, CAS are composed of active agents that behave as if responding to an internal rules structure, and their behaviors at the local level combine to create patterns of activity higher-level meta-agents. Those meta-agents can likewise be thought of as agents operating in still higher complex adaptive systems.

Adaptation, "the sine qua non[3] "of CAS (Holland, 1995, p. 8) is a feature of all CAS and Holland often relies on biological metaphors to characterize this attribute. He says that adaptation is how the "organism fits itself to its environment," and that "experience guides changes in the organism's structure… [in order to] make better use of its environment for its own ends" (p. 9). If agents act as if they were responding to an internal set of rules, then a way for them to learn is by modifying those rules in response to experience. Rules sets can thus be "viewed as hypotheses that are undergoing testing and confirmation" (p. 53). Holland's treatment goes into significant detail about how the transformation of rules might proceed, but it is sufficient for the present purpose to say that the process is recursive, based on information from the environment (feedback) that the agent is immersed in, and results in reorganizations of rules sets.

[3] "without this, nothing".

Another aspect of Holland's (1995) view of adaptation needs to be discussed, because it sheds light on the ways in which patterns emerge from the activity of individual agents. Each agent's environment is partly composed of other agents, "so that a portion of any agent's efforts at adaptation is spent adapting to other adaptive agents. This one feature is a major source of the temporal patterns that CAS generate" (p. 10). The agents of a complex adaptive system are constantly adapting to their environment, and that environment includes other adapting agents, and the net effect is the evolution of distinct patterns of activity when viewed from "one level up."

At this point the notion of adaptation is probably sufficiently defined for the purposes of this essay: adaptation is learning (e.g., changing internal rules-sets) based upon interactions that the agent has with its environment. A major component of that environment is other adaptive agents, and the patterns of activity generated by agents' individual and mutual adaptation provide the observable organizational patterns. It is also important to note that, as agents and their behaviors evolve, so does their *external* environment. Although Holland doesn't discuss this explicitly, I believe the idea that agents and their environs can be seen as being mutually adaptive provides an additional and potentially useful piece to the puzzle of CAS.

The Seven Basics: Mechanisms and Properties

These "seven basics"—the mechanisms and properties—that Holland (1995) considers common characteristics of all CAS "are not the only basics that could be selected" (p. 10) as ways of understanding the activity of complex adaptive systems. He reminds us that, as researchers, we still need to be a bit artful, choosing which characteristics will provide useful foci for particular investigations for particular purposes. "This is not so much a matter of correct or incorrect… as it is a matter of what questions are being investigated" (p. 8). At the same time, Holland intends his work to generate a model that can be used for studying *all* CAS and he writes that "all the other candidates" for mechanisms and properties that he's encountered can be "derived" from "combinations of these seven" (p. 10). The mechanisms—tagging, building blocks, and internal models—are *how CAS act* and how they adapt. The properties—diversity, flows, non-linearity, and aggregation—are *observable characteristics* of that activity and adaptation.

Mechanisms Discussion.

Tagging is the process of complex adaptive system's identifying features in its environment that are relevant in determining its future activity. A CAS selects particular features (building blocks) from all the possible stimuli in its environment as a function of a currently active set of tagging rules. Tagging rules structure agents' parsing their environments by motivating and driving selective attention. When a CAS first encounters a situation, a preexisting set of tagging rules relevant to the particular situation becomes active, and the rules specify particular things for the CAS to expect (or not expect), and to look for. In CAS, tagging is the means by which "building blocks" are extracted from experience in a "perpetually novel environment" (p. 34), and the selected building blocks are subsequently assembled into an internal model of the complex adaptive system's environment.

As has been pointed out, the building blocks mechanism refers to the complex adaptive system's means for creating and adapting useful internal models. A CAS identifies building blocks according to the operative tagging rules, it isolates those bits of information in its environment and combines and re-combines them into internal models. As an example, consider the enormous number of variations generated in the process of police artists' creating composite drawings of suspects from general sets of facial components (Holland, 1995, pp. 34-36). In the same way that composite sketches are created, complex adaptive systems combine building blocks and generate internal models of their environments.

The activity patterns of a CAS are directed by means of the third mechanism, internal models. The major functions of internal models are to enable the CAS to anticipate the consequences of future actions and to subsequently adapt its behaviors in order to become more successful at satisfying its needs. Depending on a multitude of factors, anticipation and internal models may be unconscious, as is the case in a bacterium or in automatic functions (Anderson, 1983) in learners, or very conscious as in the case of a person's using instructions from a friend to navigate in an unfamiliar city. Internal models are real-time and *transitory* representations of the more-or-less local environment of a CAS that provide it with the means for anticipating possible outcomes of its activity. Internal representations are transitory because they are continually being adapted in response to feedback from the rest of the system. In addition to adaptation of internal models, the tagging rules set that was driving the process of internal model-building may be permanently altered as a consequence of experience, and the adapted rules-set will persist in memory structures of the system for subsequent use.

Properties.

Beyond the attributes and mechanisms of CAS, Holland (1995) presents a list of four properties that further serve to characterize complex adaptive systems: diversity, flows, non-linearity, and aggregation. At this time, I think of the properties as descriptive—as a sort of a laundry list of observables that help in making the determination as to whether a system is a CAS or not. If a thing is a CAS, then one ought to be able to identify the four properties in its activity; if the properties aren't in evidence, then it's probably not a CAS. It seems likely that the properties will mature into the central foci of complexity-based approaches to learning and teaching, because for example, it's in studying the diversity and non-linearity of CAS that one is apt to locate the "lever points" (p. 97) that enable large changes from small inputs. At present however, the mechanisms alone occupy center stage in my research.

CAS and learning: Summary and assumptions

In complex adaptive systems, stored sets of tagging rules are activated based upon the environmental context and the rules cause the CAS to look for particular affordances in the current situation. As these affordances (building blocks) are identified, they combine to generate an internal model of the situation (Holland, 1995, p. 37) that enables the CAS to anticipate likely outcomes of its future activity. Real-time internal models are transitory and evolving, depending upon the changing environment and the relative fruitfulness of the activities based on the internal model. As time passes what persist are the tagging rules sets, which may also have adapted, and which will be retained by the complex adaptive system later use. The properties of non-linearity, diversity, flows, and aggregation are the ways in which CAS behave as functions of the mechanisms that drive them.

Two general assumptions of my research are: 1) that individual learners, groups of learners, or entire classrooms are complex adaptive systems; and 2) that adaptation in these CAS is synonymous with learning. As described above, an important attribute of CAS is that they are composed of agents that aggregate into meta-agents. In a classroom, one can think of individual learners as agents and groups or the entire classroom as meta-agents whose patterns of activity arise from the activity of agents (individual students) at the next level down.

Similarly, an individual learner can be considered to be a meta-agent, whose activity is the observable pattern generated by interacting agents—the agent's own, competing, internal models. To borrow a metaphor from Toulmin (1972), a learner's internal models taken together constitute a "conceptual ecology." An individual learner's conceptual ecology can be seen to possess all of the attributes, mechanisms, and properties present in Holland's (1995) model of a complex adaptive system. Learning in these systems consists of the development of real time internal models and assimilation and accommodation of feedback information into persistent tagging rules sets.

An Example of the Mechanisms

Perhaps understanding of the mechanisms will be aided by a real-world example of how they might work. Imagine that it's getting dark as you're driving in an unfamiliar part of your city with a lot of road construction—you're coming up to an intersection and preparing to make a right turn onto a cross street. As you approach the intersection, you look down the cross street in anticipation of the turn. At this point, a generic two-way street tagging rules set (TRS1) becomes active in your mind and you begin to sample your environment based on the following rules:

TRS1—Two-way streets

If it's a two-way street, then

- Expect a centerline stripe.
- Look for cars moving in two directions.
- Expect cars to be parked facing away from you on the left side of the street, and toward you on the right side.
- Look for some signs with their faces toward you, some with their backs toward you.

As you get closer to the street you realize that it is under construction and that the things you'd expected to find are not entirely in evidence. What you see is that there are three parked cars, and they're all facing you. You can only see two signs on as you look down the street, and they're both facing away from you. In addition, there are no visible stripes on the roadway and no cars moving along the street.

TRS 1 has proven to be increasingly unsuccessful in helping you decide on a course for future action. As its fruitfulness fades, a second set of tagging rules becomes active. TRS 2 is a one-way street tagging rules set:

TRS 2—One-way streets

If it's a one-way street, then

- Expect a sign announcing that the street is a one-way street.
- Look for cars moving in one direction.
- Expect all cars to be parked facing in one direction.
- Expect all signs to be facing in one direction or the other.

You immediately begin sampling the environment for these salient features and you find that the first condition is not met but now there's a car coming toward you. Again you notice the three cars on the street (all facing you), and the two signs that you can see are indeed facing away from you.

This information spontaneously combines to generate a real-time internal model that causes you to turn off your right turn signal and continue traveling up the street you're presently on. The model that has emerged looks something like this: "There's a car coming toward me, the parked cars on the street are all facing toward me, and the signs are all facing away from me. There isn't a one-way street sign, but this is a construction area, so maybe the workmen haven't gotten around to putting all of the signs up. I'm going to treat it as if it were a one-way street opposing me, and try the next street." TRS 2 has helped you to identify a set of building blocks (cars, signs, etc.) that were present in your environment these and these were put together to form a real-time internal model that enabled you to make sense of your experience.

As you approach the next cross-street, you notice a one-way street sign facing you, and several parked cars all pointing away from you on both sides of the street. You also notice one car traveling away from you and that there are no signs along the street with their backs to you. You confidently signal and turn right onto the street. While you're doing this, a new set of tagging rules presents itself and TRS 2 is amended (TRS 2.1) and fades away for the time being.

TRS 2.1

- Expect a sign announcing that the street is a one-way street.
- Look for cars moving in one direction.
- Expect all cars to be parked facing in one direction.
- Expect all signs to be facing in one direction or the other.
- Look for cars pointing the same direction and parked on both sides of the street.

In complex adaptive systems, stored sets of tagging rules are activated based on cues from the environment and they tell the CAS to look for particular affordances in the current situation. As these affordances (building blocks) are identified, they are combined to generate a tentative internal model of the situation (Holland, 1995, p. 37) that enables the CAS to anticipate likely outcomes of its future activity. Real-time internal models are transitory and evolving, depending upon the changing environment and the relative fruitfulness of the complex adaptive system's activity. What persists is the tagging rules set, which may itself adapt and which will be retained by the complex adaptive system later use.

CAS and Learning in Classrooms

Two general assumptions in this paper are: 1) that individual learners, groups of learners, or entire classrooms are complex adaptive systems; and 2) that adaptation in these CAS is synonymous with learning. As described above, an attribute of CAS is that they include agents and meta-agents. In a classroom, one can view individual learners as agents and small groups or the entire classroom as meta-agents whose patterns of activity arise from the activity of agents at the next level down. At the same time, a learner can be considered to be a meta-agent, whose activity is the outward pattern generated by some number of its interacting internal models. To borrow a metaphor from Toulmin (1972), a learner's internal models taken together constitute a "conceptual ecology"—a complex adaptive system that exhibits all of the attributes, mechanisms, and properties present in Holland's (1995) model of a complex adaptive system.

For the purposes of the current research project, I have chosen to focus on individual learners and to consider each to be a complex adaptive system. The reasons for choosing individual learners as the focus are three-fold. First, although conscious thought and decision making are not requisite criteria for CAS, it is much easier to make the connection between a model of a complex adaptive system and the system itself if at least some of the tagging rule decisions are made consciously. Second, much if not all of human behavior is goal-directed, and the use of internal models in the anticipation and acquisition of goals is well motivated. Finally, as mentioned in the introduction, the CAS model being discussed here has much in common with the current cognitive literature and research understandings of individual learners. It is my hope that in the research that follows that I can present a description of individual learning that is plausible, relatively clear, and reasonably connected to existing research on learning.

The Study

Methods and Approach

This is a pilot investigation in a study of individual and group learning as complex adaptive systems. The primary purpose of the study is to instantiate a new theoretical framework for making sense of human learning and to generate plausible means for identifying the three mechanisms (Holland, 1995) that complex systems use in adaptation. The subjects of this pilot study were nine university students, several of whom were pre-service mathematics and science teachers. Their participation was voluntary and there was a small stipend paid. The results are intended to be strictly descriptive.

The research reported here is a close study of one of the nine learners that were engaged in a networked simulation—the HubNet participatory simulation (Wilensky & Stroup, 1999; 2000) known as Gridlock. Participatory simulations are networked activities where learners act out the roles of elements in the system and observe the behavior of the system as a whole as patterns emerge from their individual behaviors. These results then become the focus of participants' discussions and analyses. Using network technology with a public display space where traffic flows and the city's streets are projected at the front of the classroom, each learner controls a stoplight and together they work toward improving the flow of traffic through the city.

When introduced to the gridlock activity, students were presented with the following scenario: The mayor of the City of Gridlock is unhappy with the traffic congestion in town and she has commissioned the class to improve the situation (Wilensky & Stroup, 2000). The goal of the activity is for the students to find ways of optimizing traffic flow for the simulated city and to report back to the mayor. Students are asked what they know about traffic flow and the learners articulate a wide range of factors that can impact complex phenomena like traffic. During the simulation, students control individual traffic lights and also call out directions and negotiate strategies with each other. In addition to an introduction to the Gridlock activity, participants received instruction in the use of concept mapping (Novak & Gowin, 1984) at the beginning of this study.

Data collection and analysis

The data for this study consisted of participants' concept maps of how city traffic flows done before and after the simulations, videotaped records of all the activity, a videotaped interview with one of the participants, and observer field notes. In addition, the entire state of the simulation, including participants' key-presses, the upfront projection, and the real-time flow of traffic, was recorded for every run of the simulation. All data has been analyzed for evidence of tagging rules, building blocks, and internal models, and for how these mechanisms evolved throughout the course of the activities. The pre- and post-activity concept maps were compared in an effort to characterize changes in participants' "traffic flow internal models." The data are intended mainly to help to paint a descriptive portrait of the three mechanisms of CAS.

Results

The results being reported here are a close study of the adaptation of one of the male participants in the simulations. Although the focus in this paper is on a single individual situated in a group learning experience a central claim of the CAS perspective on learning is that it is applicable and has value for making sense of learning at multiple levels I will return to a brief discussion of a group level discussion of CAS learning in my concluding remarks.

The table below contains excerpted transcriptions of the participant's contributions to the group discussions together with brief analyses of the meanings attributed to these statements vis-à-vis a complex adaptive systems model of learning.

The data from this study is rich, and only a small portion of the analysis of the study can be reported here.

Table A1. The left hand column is the transcription of the participant's conversations, the right hand column is the analysis.

Participant Comment	Analysis
1. One thing I've noticed about doing this--it's very hard to pay attention to what's going on at the other lights to try and see if there's any patterns about how things work... without ignoring my own light and jamming up my intersection.… Every time I look and see if there's any patterns for how the lights are going and how the traffic's flowing then I have to let go of my intersection and let it jam up…	*Building Blocks*. The participant is pointing out what he's looking at, the salient features that he's sampling in his environment: other lights, patterns, his own light. He's looking only at the simulation as it's unfolding in trying to make sense of the emergent phenomena. His tagging rules are causing him to focus closely on actual activity in a very localized region around his traffic light.
2. I'd see all the green lights were moving this way and then this way and there's a pattern there...	*Building Blocks*. The participant is telling us which components of his environment he's sampling—patterns of green lights flowing through the system.
3. My intuition's telling me you want some alternating pattern, red, you know… there's two variables each place here but you know, you want to set it up the lights to go red, green, red, green, red, green, and to alternate going through the system.	*Internal Model.* His "intuition" is really an emergent internal model about how to improve traffic flow.
4. I think the first thing we should do is every time there's a clock tick, every body switches their light. So the board's set up, right now, the vertical lines are going and the horizontal lines are stopped.	*Internal Model.* The model he's building now includes a temporal component along with the alternation of red and green lights.
5. Another participant: Is that your measure down there [pointing at Average Wait Time graph]? How the traffic is [flowing]?	*Building Blocks.* This is the first time that any of the participants has mentioned any of the graphical presentations of data in the up-front space.
6. We need some sort of time clock.	*Tagging Rule.* The participant's operative rules set is beginning to look

	for building blocks that are "better" than just the information in the traffic grid itself.
7. There need to be time scales on the graphs. So we can see whether these wave motions have anything to do with our time… between lights [switching]. It's just if there were crosshairs... we could see where that was; and we could say if these were nine second bumps or two second bumps.	*Building Blocks.* The first participant immediately seizes on the graph—putting the graphical information, timescales between lights changes, and traffic flows together.
8. I'm just trying to reconcile the average speed not changing much and the average wait time going way up. The average speed over the whole second fifty seconds is about the same as average speed over the whole first fifty seconds, but the average wait time over the second fifty seconds is like two or three times what it is over the first fifty seconds.	*Building Blocks.* The participant is comparing two different potential building blocks, and trying to make sense of the apparent conflict.
9. Yeah... there's gotta' be some sort of math about the number of cars that are stopped... that makes that iron out.	*Tagging Rules.* This is relative to the above conflict. It's important to note that now his set of tagging rules is beginning to look for mathematized building blocks to try and resolve the conflict.
10. I guess I'm thinking from the standpoint of the hypothetical person running for election (the Mayor of Gridlock). The number of people that get to work get to work at the same time in simulation one as in simulation two, but they're going to perceive that it's going slower in simulation two because they wait longer.… So if you had to choose, from the standpoint of someone wanting to get reelected… it matters definitely--you want five seconds as opposed to ten seconds, but it doesn't matter practically because the speeds are staying the same.	*Internal Model.* The participant is discussing two competing internal models for improving traffic flows. In simulation one, the time between changes in the traffic lights is five seconds. In simulation two, the time between changes is ten seconds. However, in sampling information (building blocks) from the two graphs, the average speed of the cars graph is the same whether the time between changes is five seconds or ten seconds. His internal model therefore says, the traffic will seem improve if cars wait only five seconds, because a shorter wait will be seen as better, even though the average speed of the traffic remains unchanged.
11. We are definitely getting into a problem here which is a psychological problem. What will drivers think as opposed to how fast will	*Tagging Rules.* Here we see the participant's rules sets are changing again—he's moving from sampling

they actually get to work?	graphical/mathematical building blocks to looking for *psychological* building blocks.
12. How do people actually think? I mean... I don't know, I've never really thought about this. I know that I tend to get emotionally more frustrated by being stopped. How general is that? Is that something that affects everybody? Are they really going to vote on the basis of how long they're stopped? Because from a practical standpoint you'd want to use that graph in the middle—which just gives you the average speed—you want to maximize the speed through the grid. But is that how people are going to vote?	*Building Block.* The question for the participant here is what to choose as building blocks for his internal model—strictly mathematical data, or information based on the psychological perceptions of drivers.

Affordances of CAS View of Learning

It is important to point out that Holland's (1995) complex adaptive systems model is only one of a large number of potentially useful and productive systems theoretical approaches to studying learning. For example, Hurford (1998) discusses a self-organizing critical systems perspective on conceptual change. A case can be made, for saying that much of experiential learning is self-organized learning, and systems lenses like the one provided by Camazine, et al., (2001) may provide the tools for getting at important insights into individual and group learning. There has already been a good deal of work done in characterizing learning at the level of organizations (Senge, 1995; Sterman, 2000) using systems dynamics tools and software and much more remains to be done in this direction.

The theoretical perspective presented here has much in common with learning theories based on individual cognition. It includes elements of conceptual change models (Demastes, Good, & Peebles, 1996; Hurford, 1998; Strike & Posner, 1992; Vosniadou & Brewer, 1987) and cognitive structures (di Sessa, 1993). A CAS view of learning also sees learners as changing their conceptual structures and it extends the former perspectives, situating conceptual changes in persistent tagging rules sets. A CAS view of learning is like constructivist perspectives (Cobb, 1994; Piaget, 1929/1951; Steffe & Gale, 1995), in that learners are actively constructing knowledge, but it points to the mechanisms (tagging rules, building blocks, and internal models) and attends to the emergence of structures (Piaget, 1968/1970), by which constructivist learning is believed to occur. The CAS view of learning meshes with schema-theoretic perspectives (Derry, 1996; Schank, 1996), the tagging rules *are* schemata (Holland, 1995, p. 90). At the same time the perspective presented in this paper adds a layer to schema theories—the real-time and transitory internal models that constantly change are seen to continuously and dynamically update as functions of experience and the local environment. The combination of internal models and experience feeds back to persistent schemata (tagging rules sets) and adapts them. The CAS view of learning also has something to add to expert-novice perspectives (Bransford, Brown, & Cocking, 2000; Reiner, Slotta, Chi, & Resnick, 2000) because, as discussed above, it provides a way of thinking about mechanisms and trajectories for learners' progressing from novice to expert.

This approach is different from all of the above though, in that it also supports a perspective relative to *learning at the group level*. An attribute of dynamical systems is that they are self-similar—they possess features that are either the same (e.g., a Koch snowflake) or very nearly the same (e.g., fractals) over a wide range of scales. This is a very important affordance of a CAS perspective on learning, the same lens that has been applied here is can be applied to learning at the group level. The tagging rules, building blocks, and internal models seen at the level of the individual were identified in this research study at the group level as well. A complex adaptive systems analysis of group learning is will be reported on in future work

In this group sense, a CAS view offers an alternative framework for Cobb's group-level symbolic interactionism (Cobb, 1994)—the CAS lens is a semi-mathematized structure for thinking about how ideas, learning, and knowledge emerge and adapt in classrooms. Beyond Cobb's view of the social space, CAS learning is congruent with the proposed "mathematics structuring the social space" (MS3) perspective (Stroup, Ares, and Hurford, 2005). MS3 considers activity in classrooms as being generally structured by mathematical understandings, a CAS perspective on classrooms brings a particular (dynamical systems) lens to MS3 and gets us one step closer to mathematized models for learning at a wide range of scales.

An Extension of Holland's Model and Connections to the Literature on Learning

Tagging, according to John Holland (1995) is a mechanism for identifying features in the environment of a CAS that are salient in determining its future activity. A CAS selects salient features (building blocks) from all the possible inputs in its environment as a function of a currently active set of tagging criteria. Tagging structures agents' parsing their environments by driving selective attention. When a CAS first encounters a particular situation, an existing set of tagging criteria relevant to the situation becomes active, and the tags specify things for the CAS to expect (or not expect), and to look for. In CAS the tagging mechanism is the means by which "building blocks" are rendered from experience in a "perpetually novel environment" (p. 34), and the building blocks are subsequently assembled into a real time internal model of the complex adaptive system's environment.

As envisioned by Holland, tagging is a universal mechanism of all complex adaptive systems, however I believe there is something very different and important between the behaviors of ants (Gordon, 1995), bees and birds (Camazine, et al., 2002), or computer programs (Holland, 1995, 1998), and human behaviors. I propose a more sophisticated and specifically human mechanism that extends Holland's (1995) tagging[4]. I refer to this mechanism as "persistent internal models" (PIMs) and distinguish them from the "real-time internal models" (RIMs, these are the same as Holland's "internal models" mechanism) that complex adaptive systems form in the course of activity. Persistent internal models are more than just tagging rules sets—I envision them as whole conceptual structures possessing both declarative and procedural information that the learner keeps in long-term memory and that are activated by the learner's context.

Persistent internal models have the same function as tagging mechanisms—they direct attention and enable the acquisition of building blocks for composition into real-time internal models. PIMs also store knowledge and experience for future use. At the same time, they are more than just rules-sets: what I am calling PIMs (in this CAS

[4] I have discussed elsewhere the dangers of "co-optation and renaming" of terms and in the interests of brevity, omitted that discussion here.

context) have elsewhere been called schemata (e.g., Piaget, 1968, pp. 81-83), conceptual structures (Strike & Posner, 1992; Vosniadou & Brewer, 1987), cognitive structures (diSessa, 1993), mental models (Gentner & Stevens, 1983), scripts (Abelson & Schank, 1977) and simply "models" (Lesh & Doerr, 2003). While I recognize that these ideas are not necessarily identical to each other and that my sense of PIMs is derivative of all of them, I use these examples to help illustrate my conception. I have elected to retain this new term, persistent internal model, as being more congruent with Holland's (1995) CAS model and at the same time trying to avoid terms that "[carry] too much unintended conceptual baggage" (Lesh & Doerr, 2003, p. 8).

Particular PIMs, persistent conceptual structures, become active based on the learner's current context. Then they help the learner to form a real time internal model (RIM) of the context, and that real time model enables the learner to make sense of his or her environment and to anticipate likely results of activity and behaviors. PIMs help to structure RIMs by enabling the learner to select building blocks (bits of information) from its situation and then to assemble them, by substituting them into appropriate places in the existing PIM framework (this is very similar to Schank's notion of scripts, but see below). The old PIM framework, having been thus restructured by information in the learner's immediate environment, becomes the real time internal model. As the name implies, RIMs are constantly being updated and revised as time proceeds. As the situation changes, alternative PIMs may become active in response the changes and the existing RIMs are replaced with new ones. The previous PIMs may have been altered to some degree based upon the experience, and the restructured PIM is what gets stored for the future.

As I alluded to above, what I am describing here has a lot in common with Abelson & Schank's (1977) scripts. It also is derivative of work on conceptual change theories and Piaget's notions of assimilation and accommodation. What is different, and I claim what adds value to a CAS learning approach is the inclusiveness of the CAS model. The CAS model is a whole picture of learning that can parsimoniously combine many current learning models into a larger and more cohesive package. Although a detailed account of the particulars is beyond the scope of the current paper, much of my motivation for the development of a CAS model of learning comes from the fact that such a model has the potential for connecting previous models of learning and for filling in the spaces between them. It is in the interests of this overall project that this CAS learning model is being formulated and some of the first steps from theory-borrowing to theory-building are being made.

Appendix References

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